Math 130 – Spring 2015 – Boyle –Exam 3

- NO CALCULATORS OR ELECTRONIC DEVICES ALLOWED.
- Use a separate answer sheet for each question.
- Give your pledge on page 1 only, covering the whole test.
- Draw a box around a final answer to a problem.

1. (14 points)
Let f be the function $f(x) = x^2 - 8\ln x$ with domain $[1, 10]$.

(a) (4 pts) What properties of f and its domain guarantee that f will assume maximum and minimum values?

(b) (10 pts) What are the maximum and minimum values assumed by f on its domain?

2. (10 points)
Find the equation of the tangent line to the curve $4e^{2x} - y^2 = 0$ at the point $(0, 2)$.

3. (15 points)
Let f be the function with domain $[0, 2]$ defined by $f(x) = \sqrt{2x+1}$.

(a) (7 pts) Compute the left endpoint Riemann sum estimate $\sum_{i=1}^{4} f(x_{i-1})\Delta x$ of $\int_{x=0}^{2} f(x)\,dx$ when $n = 4$. (Do not simplify the expression you obtain from the definition.)

(b) (5 pts) Draw the graph of f and the rectangles corresponding to this Riemann sum.

(c) (3 pts) Is this Riemann sum greater or smaller than $\int_{x=0}^{2} f(x)\,dx$?

4. (14 points)
Let f be the function on $[0, 4]$ defined by $f(x) = (2x+1)^{1/4}$. Let R be the "region under the curve", i.e. the set of points (x, y) such that $0 \leq x \leq 4$ and $0 \leq y \leq f(x)$. Let S be the solid of revolution obtained by rotating R about the x-axis.

What is the volume of S?

**** THERE ARE MORE PROBLEMS ON THE OTHER SIDE. ****

5. (18 points)

(a) (8 pts) Compute the average value of the function $f(x) = \sec^2(x)$ over the interval $[0, \pi/4]$.

(b) (10 pts) Evaluate the definite integral

$$\int_{x=\pi/4}^{\pi/2} \sqrt{\sin x}\, \cos x \, dx$$

6. (14 points)

Let $s(t)$ be the position of a certain object at time t. Suppose its velocity at time t is e^{2t}, and suppose $s(0) = 1$.

What is the position of the object at time $t = 3$?

7. (15 points) According to Poiseuille's laws, the velocity v of blood in a blood vessel is given by $v(r) = k(R^2 - r^2)$, where R is the (constant) radius of the blood vessel, r is the distance of the flowing blood from the center of the blood vessel, and k is a positive constant.

Given R, let $Q(R)$ be the total blood flow (in milliliter per minute) in the vessel. For n a positive integer, $Q(R)$ is approximated by a sum

$$\sum_{i=1}^{n} v(r_i) 2\pi r_i \Delta r$$

in which $\Delta R = R/n$ and $r_i = i\Delta r$. As n goes to ∞, the sum converges to $Q(R)$.

(a) (5 pts) Write a definite integral which equals $Q(R)$.

(b) (10 pts) Compute the definite integral.