Fall 2014 - Math 463
Complex Variables for Scientists and Engineers
Homework #2 - Due Tues. Sept. 16 **in class**

1. Give a polar form for $z = (2 - 2i)^3$. What is its principal argument $A(z)$?

2. Find all the roots (first in exponential form, then give the real part and imaginary part) of $(-8)^{1/3}$.

3. Describe and sketch the set of points determined by the following conditions (one graph for each):
   (a) $|z - 3i| \geq 1$
   (b) $z \neq 0$ and $-\pi < \mathrm{Arg}(z) < \pi$
   (c) $\mathrm{Re}(z) < 1/2$
   (d) $\mathrm{Im}(z) = 1$

4. For each set in Exercise 3, describe and sketch the interior, the closure and the boundary.

5. Which sets in Exercise 3 are open? Which sets are closed?

6. (a) Write the function $f(z) = z^2 + z + 1$ in the form $f(z) = u(x, y) + iv(x, y)$.
   (b) Write the function $f(z) = \dfrac{z}{\bar{z}}$ in the form $f(z) = u(r, \theta) + iv(r, \theta)$.
   (c) Suppose that $f(x + iy) = x^2 - y + ixy$. Write $f(z)$ in terms of z (Hint: use the relations $\mathrm{Re}(z) = \frac{z+\bar{z}}{2}$ and $\mathrm{Im}(z) = \frac{z-\bar{z}}{2i}$)

7. We examine some properties of the exponential function $f(z) = e^z$. If $z = x + iy$ (with x and y real numbers), then we define $e^z = e^x(\cos y + i \sin y)$.
   (a) Are there any complex number z such that $f(z) = 0$? (justify your answer)
   (b) Describe the images under f of each of the following lines:
   (a) $\mathrm{Im}(z) = 1$ (b) $\mathrm{Re}(z) = 1$ (c) $\mathrm{Im}(z) = \mathrm{Re}(z)$.
   (c) Sketch the region onto which the set of points satisfying $0 \leq \mathrm{Re}(z) \leq 1$, $0 \leq \mathrm{Im}(z) \leq 1$ is mapped by the transformation $f(z) = e^z$.

From the text do the following (e.g. 14: . . . lists problems from Section 14).

- Sec. 14: 1bd,3,6,8b
  (Note: the answer listed for (b) should have been listed instead for (c).)
- Sec. 18: 10
- Sec. 24: 1acd