Fall 2014 - Math 463
Homework #3 - Due **in class Tues. Sept. 23**

1. Sketch the region onto which the sector $r \leq 1$, $0 \leq \theta \leq \pi/4$ is mapped by the transformation $f(z) = z^2$.

2. For each of the following, find the image of S under the transformation $w = f(z)$.

   (a) $f(z) = (1+i)z$; S is the line $y = 2x + 1$.
   (b) $f(z) = 1/z$; S is the circle $|z| = 2$.

3. Find the image of the circle $|z - z_0| = R$ under the transformation $f(z) = iz - 2$.

4. Show that the image of the vertical line $\text{Re}(z) = 1$ under the transformation $f(z) = 1/z$ is a circle of radius $1/2$, centered at $z_0 = 1/2$.

5. Consider the limit $\lim\limits_{z \to 0} \left(\dfrac{\overline{z}}{z} \right)^2$

   (a) What value does the limit approach as z approaches 0 along the real axis?

   (b) What value does the limit approach as z approaches 0 along the imaginary axis?

   (c) Does $\lim\limits_{z \to 0} \left(\dfrac{\overline{z}}{z} \right)^2$ exists? Explain.

6. Show that the function $f(z) = \text{Arg}(z)$ is discontinuous at $z = -1$.

7. Problem 1, page 70 (Sec. 24)

8. Sec. 26: 1a,1c,6