Fall 2014 - Math 463
Complex Variables for Scientists and Engineers
Homework #1 - Due Tuesday Sept.9 **in class**

1. (a) Give the real part, imaginary part and modulus of the following complex numbers:

$$-3+9i, \quad -2i, \quad 5, \quad \overline{-5i+3}, \quad \overline{1+2i+\overline{(3-i)}}$$

   (b) Let $z_1 = 3+2i$ and $z_2 = 4-2i$. Give the real part and imaginary part of the following complex numbers:

$$z_1+z_2, \quad z_1-z_2, \quad z_1 z_2, \quad z_1^{-1}, \quad \frac{z_2}{z_1}.$$

2. Verify that each of the two numbers $z = 1+i$ and $z = 1-i$ satisfies the equation $z^2 - 2z + 2 = 0$.

3. Plot the following points on a graph:

$$z_3 = -3i, \quad z_4 = 1+2i, \quad \overline{z_4}, \quad z_3+z_4, \quad 2z_4.$$

4. Describe and sketch the set of points determined by the following conditions (one graph for each):

   (a) $|z-4+3i| = 3$
   (b) $|z| \leq 3$
   (c) $\mathrm{Im}(z) = 3$
   (d) $\mathrm{Re}(\overline{z}+2+i) = 1$

5. Show that for any complex number z we have

   (a) $\mathrm{Re}(iz) = -\mathrm{Im}(z)$
   (b) $|\overline{z}| = |z|$

6. Writing $z = x+iy$, show that the set of all complex numbers satisfying

$$z^2 + \overline{z}^2 = 2$$

   is an hyperbola.