Math 461 - Linear Algebra for Scientists and Engineers

MATLAB Assignment #2

More matrix operations:

- If A is a matrix with real entries, then `A'` is the transpose of A (if A has complex entries then `A'` is the conjugate transpose).
- If A is a square invertible matrix, then `inv(A)` is the inverse of A.
- If A is a square matrix, then `det(A)` is the determinant of A.
- If A is a matrix then `max(A)` gives a row vector whose entries are the maxima of the entries in each column of A. `max(max(A))` gives the maximum of the entries of A. Other useful operations include `sum`, `min` and `prod` which can be used to find the sum, minimum and product of the entries of a matrix.

Checking that two large matrices are equal: If you want to check that two large matrices A and B are equal it can be tedious to compare each entry (a 10×10 matrix has 100 entries). A faster way is to look at the difference $B - A$ and see if it is zero. To do this, it is best to use the command

```
>> max(max(abs(B-A)))
```

which will print out the entry of $B - A$ with the largest absolute value.

Note that because of roundoff error, two matrices might not be quite equal, even though they should theoretically be equal (and would be if the computer could do exact arithmetic). For example `A-inv(inv(A))` will generally not be 0 even though it should (try it for a random 4×4 matrix). This means that when $B - A$ is very small compared to A and B we will interpret the result as being 0.

In the following problems, you should suppress all unnecessary output by the use of the semicolon.

Problem 1. Generate two 10×10 matrices A and B with random entries (use the semicolon to suppress the output). Compute $C = AB$, $D = B^T A^T$ and $E = B^{-1}A^{-1}$ (again, use the semicolon to suppress the output). Check that $D = C^T$ and $E = C^{-1}$ (using the `max` command as explained above).

Problem 2. Generate a 6×6 matrix A with random entries. Row reduce $[A \quad I_6]$ to reduced echelon form and extract from this A^{-1}. (You can generate the augmented matrix as `[A eye(6)]`. You can extract the 7th to 12th columns of a matrix D with the command `D(:,7:12)`).

Compare your result with `inv(A)`.

Problem 3. Generate a random 7×7 matrix A and a random vector $\vec{b}$ in $\mathbb{R}^7$. Solve the equation $A\vec{x} = \vec{b}$ using two different methods:

1. using the command `rref` to row reduce the appropriate augmented matrix (you can extract the 8th column of a matrix D with the command `D(:,8)`),
2. computing directly $A^{-1}\vec{b}$.

Compare your results.

Problem 4. Generate two random 7×7 matrices A and B and check whether or not each of the following identities holds:

1. $\det(AB) = \det(A)\det(B)$
2. $\det(A + B) = \det(A) + \det(B)$
3. $\det(A^{-1}) = 1/\det(A)$
4. $\det\left(\begin{bmatrix} A & 0 \\ 0 & B \end{bmatrix}\right) = \det(A)\det(B)$ (where the 0 denotes the 7×7 zero matrices).

Problem 5. Generate a random 4×5 matrix A. Compute $\det(A^TA)$ and $\det(AA^T)$. Repeat for several random matrices.

What can you say about A^TA and AA^T when A has more columns than rows?

Problem 6. Compute the determinant of the following matrices

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 \\ 1 & 2 & 3 & 4 \end{bmatrix}, \quad \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 \\ 1 & 2 & 3 & 3 & 3 \\ 1 & 2 & 3 & 4 & 4 \\ 1 & 2 & 3 & 4 & 5 \end{bmatrix}.$$

Use the results to guess the determinant of the $n \times n$ matrix below:

$$\begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ 1 & 2 & 2 & \dots & 2 \\ 1 & 2 & 3 & \dots & 3 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 2 & 3 & \dots & n \end{bmatrix}$$

Confirm your guess by using row operations to evaluate that determinant.

Problem 7. Let `V` be a random 10×10 matrix and set

```
>> U=eye(10)+1000*triu(V,1)
```

The matrix U is upper triangular with all ones in the diagonal, so its determinant should be 1. Verify that `det(U)` returns 1. In theory, $\det(U^T) = \det(U) = 1$ and $\det(UU^T) = \det(U)\det(U^T) = 1$. Use Matlab to calculate these quantities. You will probably get quite different answers. This is the reason computers cannot be trusted. Computing a determinant involves so many arithmetic operations that tiny roundoff errors can accumulate to become very significant. Though computers usually give the right answer, you should always have a healthy disrespect for a computer's answer.