

**MATLAB Assignment #4**

**Eigenvalues and Eigenvectors** If  $A$  is a square  $n \times n$  matrix, then the command `eig(A)` will produce a vector whose entries are the  $n$  eigenvalues of  $A$  (including multiplicities). If 2 is an eigenvalue, you can then find the eigenvectors for this eigenvalue with the command `null(A-2*eye(n))`. You can also use

```
>> [V D] = eig(A)
```

which will produce a diagonal matrix  $D$  whose diagonal entries are the eigenvalues of  $A$  and a  $n \times n$  matrix  $V$  whose columns are the corresponding eigenvectors. In particular, you have  $AV = VD$ . If  $A$  is diagonalizable, then the columns of  $V$  are a basis of  $\mathbb{R}^n$  and  $V$  is invertible.

**Other operations** To find the characteristic polynomial of a matrix  $A$ , use the command `poly(A)` which will return a vector of coefficients of the characteristic polynomial. The command `roots(poly(A))` will give you the roots of the characteristic polynomial. It is less accurate and more time consuming than the command `eig(A)`.

The command `real(A)` finds the real part of a matrix (or a vector)  $A$ . The command `imag(A)` finds the imaginary part of a matrix (or a vector)  $A$ .

**Problem 1.** Generate a random  $6 \times 6$  matrix and calculate the following quantities: the product of the eigenvalues, the characteristic polynomial and the determinant of the matrix. (recall that the command `prod` gives the product of a vector's entries).

**Problem 2.** Diagonalize the following matrix:

$$A = \begin{bmatrix} -6 & 4 & 0 & 9 \\ -3 & 0 & 1 & 6 \\ -1 & -2 & 1 & 0 \\ -4 & 4 & 0 & 7 \end{bmatrix}$$

Give an invertible matrix  $P$  and a diagonal matrix  $D$  such that  $A = PDP^{-1}$  (check your answer by computing `max(max(abs(P*D*inv(P)-A))`)).

**Problem 3.** Let  $T$  be the transformation whose standard matrix is given below. Find a basis  $\mathcal{B}$  for  $\mathbb{R}^4$  with the property that  $[T]_{\mathcal{B}}$  is diagonal

$$\begin{bmatrix} 15 & -66 & -44 & -33 \\ 0 & 13 & 21 & -15 \\ 1 & -15 & -21 & 12 \\ 2 & -18 & -22 & 8 \end{bmatrix}$$

**Problem 4.** Use Matlab to find a factorization of the matrix

$$A = \begin{bmatrix} 1 & -.8 \\ 4 & -2.2 \end{bmatrix}$$

in the form  $A = PCP^{-1}$  where  $C$  is of the form  $\begin{bmatrix} a & -b \\ b & a \end{bmatrix}$  and  $P$  is an invertible matrix with real coefficients.

**Problem 5.** Consider a discrete dynamical system  $\vec{x}_{k+1} = A\vec{x}_k$  with

$$A = \begin{bmatrix} 1.1 & .2 \\ .6 & .8 \end{bmatrix}$$

The following commands create a matrix  $T$  whose columns are the vectors  $\vec{x}_0 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}$ ,  $\vec{x}_1 = A\vec{x}_0$ ,  $\dots$ ,  $\vec{x}_{15} = A^{15}\vec{x}_0$ :

```
>> x=[1 ; -4]
>> T=x
>> for j=1:15
    x=A*x;
    T=[T x]
end
```

You can now use the command

```
>> plot(T(1,:),T(2,:),'or'), grid
```

to plot the vectors columns of  $T$  (the 'or' produces a red (r) circle (o) for each point of the trajectory).

By default each new use of the command `plot` will erase the previous graph. So if you want to plot other trajectories on the same graph, you have to prevent that from happening by using the command `hold on` before the second `plot` command. The command `hold off` can be used when you are ready to start a new plot.

Plot, on the same graph, the first 10 points of the trajectories of the dynamical system  $\vec{x}_{k+1} = A\vec{x}_k$  with the matrix  $A$  given above, and for the following initial vectors

$$\vec{x}_0 = \begin{bmatrix} 1 \\ -4 \end{bmatrix}, \quad \vec{x}_0 = \begin{bmatrix} -1 \\ 4 \end{bmatrix}, \quad \vec{x}_0 = \begin{bmatrix} 2 \\ -5 \end{bmatrix}, \quad \vec{x}_0 = \begin{bmatrix} -2 \\ 5 \end{bmatrix}.$$

(you should have 4 trajectories on your graph - you should use different shape/color for each trajectory by replacing 'or' with '\*g' or '+b'...). Remember to print your graph before closing it.

What can you guess about the nature of the dynamical system by looking at the graph (attractor, repeller or saddle point)? Check your guess by computing the eigenvalues of  $A$ .

**Problem 6.** Repeat Problem 5 with the dynamical system  $\vec{x}_{k+1} = A\vec{x}_k$  associated with the matrix

$$A = \begin{bmatrix} .8 & .5 \\ -.1 & 1 \end{bmatrix}$$

and the same initial vectors  $\vec{x}_0$ . You will get a nicer picture by plotting more points in each trajectories (say 50 points instead of 10).