Math 461 - Linear Algebra for Scientists and Engineers

MATLAB Assignment #5

Inner Product The inner product of two vectors `u` and `v` can be computed (using the definition of the inner product) with the command `u'*v` (or `v'*u`). The length of `u` is `norm(u)` (or `sqrt(u'*u)`). In particular, given a vector `u`, you can normalize it with the command `u/norm(u)`.

QR decomposition If A is an $m \times n$ matrix, the Matlab command

```
>> [P S] = qr(A)
```

will return an $m \times m$ orthogonal matrix P and an $m \times n$ upper triangular matrix S so that $A = PS$. If A has rank n, then the first n columns of P will be an orthogonal basis for the column space of A and the last $m-n$ columns will be an orthogonal basis for its orthogonal complement,. The last $m-n$ rows of S will be zero.

For the usual QR-decomposition given in the textbook, A must have rank n, Q is the first n columns of P and R is the first n rows of S.

So the `qr` commeand is more general since it applies to any matrix and gives more information. However, if A has rank n, then the command

```
>> [Q R] = qr(A,0)
```

will give the QR-decomposition of A in the sense seen in class.

Other commands. If A is a matrix, then `orth(A)` gives a matrix whose columns form an orthonormal basis for the column space of A. Also, we recall that `null(A)` gives a matrix whose columns form an orthonormal basis for the Null space of A.

Problem 1. Let A be a 5×5 random matrix and let $B = A^T A$ (note that the entries of the matrix B are symmetric with respect to the diagonal. Such a matrix is called a symmetric matrix). Find a basis of eigenvectors for the matrix B, and check that this basis is orthogonal.

Problem 2. Use the Gram-Schmidt process to produce an orthogonal basis for the column space of

$$A = \begin{bmatrix} -10 & 13 & 7 & -11 \\ 2 & 1 & -5 & 3 \\ -6 & 3 & 13 & -3 \\ 16 & -16 & -2 & 5 \\ 2 & 1 & -5 & -7 \end{bmatrix}$$

Problem 3. Let A be a 4×4 random matrix with rank 2 (check that its rank is 2). Let b be a random vector in $\mathbb{R}^4$.

Check that the system $Ax = b$ is inconsistent and then find a least squares solution x_0 of $Ax = b$ (is this solution unique?).

Compute the error vector $b - Ax_0$. Check that this error vector is perpendicular to the column space of A.

Compute, the error is the length $||b - Ax_0||$. Check that this error is minimized for the least squares solution by computing $||b - Ax||$ for several random vectors x and seeing that it is larger than the error.

Problem 4. Find the QR-decomposition of the matrix A from Problem 3, and check that $A = QR$ and that Q is an orthogonal matrix (check that $Q^T = Q^{-1}$).

Problem 5. Section 6.6, problem #11 (page 374)

Note: If `x` is a vector, then the command `cos(x)` returns a vector of the same size whose entries are the cosine of the entries of `x`. Also, the command `x.^k` returns a vector whose entries are the k-power of the entries of `x`.

Problem 6. Section 6.6, problem #13 (page 375)