1. Let $\mathbf{r}(t) = t\mathbf{i} + \frac{2\sqrt{2}}{3} t^{3/2} \mathbf{j} + \frac{1}{2} t^{3/2} \mathbf{k}$ be a parametrization of a curve.
   (a.) Find the length of the portion of the curve that lies between $\mathbf{r}(0)$ and $\mathbf{r}(1)$.
   (b.) Find the tangential and the normal components of the acceleration vector.
   (c.) What is the radius of curvature at the point $\mathbf{r}(1)$?

2. Let $z = e^{-y} \cos x$. Show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$.

3. Consider the function $f(x, y) = 4 - x^2 + 3y^2 + y$.
   (a.) Find the direction in which $f$ increases most rapidly at the point $(-1, 0)$. What is the maximum directional derivative of $f$ at this point?
   (b.) Let $S$ be the surface described by the equation $z = f(x, y)$ where $f(x, y)$ is given above. Find an equation for the plane tangent to $S$ at the point $(-1, 0, 3)$.

4. Let $z = x^2 + y^2$ and $x = r \cos \theta, y = r \sin \theta$. By using the Chain Rule prove that compute $\frac{\partial z}{\partial r} = 2r$ and $\frac{\partial z}{\partial \theta} = 0$. 

Spring 2009: Math 241 (Section 02); Practice EXAM 2