1. Consider the vector field
\[ \vec{F}(x, y, z) = yz^2 \hat{i} + xz^2 \hat{j} + [2xyz + 2\cos(2z)]\hat{k}. \]

(a.) Prove that \( \vec{F} \) is a conservative vector field and find a potential function \( f \) for \( \vec{F} \).
(b.) Determine \( \text{curl} \vec{F} \).
(c.) Evaluate the following line integral
\[ I = \int_C \vec{F} \cdot d\vec{r}, \]
where \( C \) is the oriented curve parametrized by \( \vec{r}(t) = \cos(\pi t^3)\hat{i} + t^{10}\hat{j} + \frac{\pi}{2} \frac{t}{1+t^4}\hat{k} \) and \( -1 \leq t \leq 1 \).

2. Let \( R \) be the region of the plane bounded by the curves \( xy = \pi/2 \) and \( xy = \pi \), \( y(2-x) = 2 \) and \( y(2-x) = 4 \). Compute the following double integral
\[ \iint_R y\cos(xy) dA, \]
by using the following change of variables: \( x = \frac{2u}{u+v} \), and \( y = u + v \).

3. Let \( S \) be the portion of the surface \( z = xy \) that is inside the cylinder \( x^2 + y^2 = 1 \).
(a.) Find a parametrization of \( S \).
(b.) Find the area of the surface \( S \).

4. Express and evaluate the following integral in cylindrical coordinates:
\[ \iiint_D (z^2 + 1) dV, \]
where \( D \) is the solid region bounded below by the upper nappe of the cone \( z^2 = 3x^2 + 3y^2 \) and above by the sphere \( x^2 + y^2 + z^2 = 4 \).