HW1, due Wednesday, February 25
Math 404, Spring 2014
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1. Is the polynomial $f(x) = x^5 + x + 1$ in $\mathbb{Q}[x]$ irreducible? If so, prove it. If not, write it as a product of monic irreducible elements of $\mathbb{Q}[x]$.

2. Suppose L/F is a field extension and p and q are two non-zero elements of $F[x]$. Let d be the monic greatest common denominator of p and q in $F[x]$. Let d' be the monic greatest common denominator of p and q regarded as polynomials in $L[x]$. Show that, in fact, $d = d'$.

3. Show that the polynomial $p(x) = x^5 + x^2 + 1 \in \mathbb{F}_2[x]$ is irreducible.

4. Let $L = \mathbb{F}_2[x]/(p)$ where $p = x^5 + x^2 + 1$ is the polynomial in Problem 3. Write α for the class of x in L. Answer the following questions about L.

(1) What is $[L : F]$?
(2) How many elements does L have?
(3) What is the multiplicative inverse of $\alpha^2 + 1$ in L?
(4) Are there any field extensions E of $\mathbb{F}_2$ contained in L besides L itself and $\mathbb{F}_2$?