

Math 406, Spring 2021

HW09, due Wednesday, May 5 at 5pm ¹

Reading: Read Chapters 8 and 9 of Crisman's text.

Graded Problems: Work the following problems for a grade. Turn them in on Gradescope.

Some problems are taken from the Online Version of Crisman's text:

<http://math.gordon.edu/ntic/>

Each problem is worth 25 points.

1. Let p be an odd prime and let k be a positive integer. Suppose r is a primitive root modulo p^k . Show that either r or $r + p^k$ is a primitive root modulo $2p^k$.

2 (Crisman 3.6.18). Suppose (x, y, z) is a primitive Pythagorean triple with x odd and $x, y, z > 0$. We already saw that this implies that y is even. Show that, in fact, y is divisible by 4.

3 (Crisman 3.6.22). Show that

$$\gcd(x, y)^2 = \gcd(x^2, xy, y^2).$$

4 (Crisman 16.8.8). Suppose p is an odd prime. Show that

$$\sum_{a=1}^{p-1} \left(\frac{a}{p} \right) = 0.$$

¹This version created Tuesday 27th April, 2021 at 21:43.