

**Math 602 Term 2 2012 at UMD**

**Homework 1**

**Due Friday, February 1.**

Read the appendix on Category Theory in Weibel up through A.3 and work the following problems.

- (1) A.1.1 on page 419
- (2) A.1.2 on page 419
- (3) A.1.3 on page 420
- (4) A.1.4 on page 420

**Problem 5.** Let  $F$  be a field and let  $\mathbf{Vect}_F$  denote the category of finite dimensional vector spaces over  $F$ . If  $V$  is an object in  $\mathbf{Vect}$  let  $D(V)$  denote the dual of  $V$ . That is  $D(V) = \text{Hom}_{\mathbf{Vect}}(V, F)$ . Explain how to define  $D$  on morphisms in  $\mathbf{Vect}$  so that  $D$  gives rise to a functor from  $\mathbf{Vect}$  to  $\mathbf{Vect}^{\text{op}}$ . Then explain why  $D : \mathbf{Vect} \rightarrow \mathbf{Vect}^{\text{op}}$  is actually an equivalence of categories.