DYNAMICAL SYSTEMS FROM ALGEBRAIC GROUPS

SEAN KELLY

Abstract. In its most general form, a (discrete) dynamical system is simply a set S and a self-map $\varphi : S \to S$. The object of study is the behavior of the iterates of the map, φ^n; in particular we want to know about the orbits of points of S. If φ is a rational map on the Riemann sphere, a deep theorem of Sullivan essentially classifies the possible behaviors of φ.

The focus of this talk will be dynamical systems $(\varphi, \mathbb{P}^1_K)$ which fit into a diagram of the form

$$\begin{array}{ccc} G & \longrightarrow & G \\ \downarrow & & \downarrow \\ \mathbb{P}^1_K & \xrightarrow{\varphi} & \mathbb{P}^1_K \end{array}$$

where G/K is an algebraic group of dimension 1. These dynamical systems are unusual in that (over $\mathbb{C}$) their Julia sets are smooth instead of fractals. If we let $K = \mathbb{Q}$ or $\mathbb{Q}_p$, we can ask arithmetic questions:

How many K-rational points are there with finite (forward) orbit?

If $K = \mathbb{Q}$, how many integral points are there?

Is there a local to global principle for dynamical systems?

The lack of algebraic structure in a dynamic system means these questions remain open, and are potentially intractable with current methods.

REFERENCES

[1] Beardon, A.F. Iteration of Rational Functions, Springer, 1991.

[2] Milnor J.W., *On Lattès maps.* Stony Brook IMS Preprint n.2004/01, Math. ArXiv DS/0402147.

[3] Silverman, J.H. The Arithmetic of Dynamical Systems, Springer, 2007.

[4] Silverman, J.H. The Arithmetic of Elliptic Curves, 1st Edition, Springer, 1986. Signed.

Date: April 1, 2010.