Homework 7 – due 03/26/08

Math 601

27. Dummit-Foote, 13.2, #8.

28. Dummit-Foote, 13.4, #5,6.

29. (5 points) Let R be any domain and $f \in R[X]$ a monic polynomial of positive degree. Show that the division algorithm works, in other words, show that for any $g \in R[X]$ we can write

$$g = qf + r$$

for $q, r \in R[X]$ and $r = 0$ or $\deg r < \deg f$. (We used this principle in class, during our proof of the existence of algebraic closures.)

30. Let K denote a splitting field for $X^8 - 2$ over $\mathbb{Q}$. Find $[K : \mathbb{Q}]$.

31. Let K be a field. Show that 0 is the intersection of the maximal ideals in $K[X_1, \ldots, X_n]$.

32. Let L/K be an extension field, and $a \in L$ an algebraic element over K whose minimal polynomial has odd degree. Show that $K(a) = K(a^2)$. Can you generalize this?