38. Let $K$ be a field of characteristic $p > 0$, and $a \in K$. Prove that if $X^p - X - a$ has no root in $K$, it is irreducible in $K[X]$.


42. Let $p$ be an odd prime, $\xi$ a primitive $p$th root of 1 in $\mathbb{C}$ and $K = \mathbb{Q}(\xi)$. We know that $\text{Aut}(K) = (\mathbb{Z}/p\mathbb{Z})^\times \cong \mathbb{Z}/(p - 1)\mathbb{Z}$. Thus $K$ has exactly one subfield $M$ with $[M : \mathbb{Q}] = 2$. Show that $M = \mathbb{Q}(\sqrt{\pm p})$ according as $p \equiv \pm 1 (4)$.

43. Let $E/F$ be a finite extension.
   (i) Prove that $E/F$ is separable if and only if there exist $[E : F]$ distinct $F$-embeddings of $E$ into $\overline{F}$.
   (ii) Show that if $E' \supset E \supset F$ are finite extensions, with $E'/E$ and $E/F$ separable, then $E'/F$ is separable.
   (iii) Deduce that if $E_{\text{sep}}$ is the subset of elements in $E$ which are separable over $F$, then $E_{\text{sep}}$ is a subfield of $E$ containing $F$. 