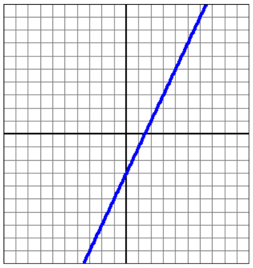
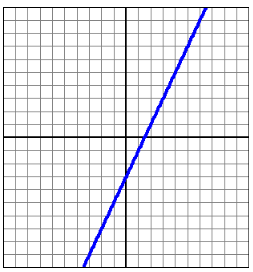
Precalculus 115, section 2.4 Average Rate of Change

notes by Tim Pilachowski

Example A-1: Given $f(x) = 2x - 3$, find the *average rate of change* over the interval [– 1, 3].



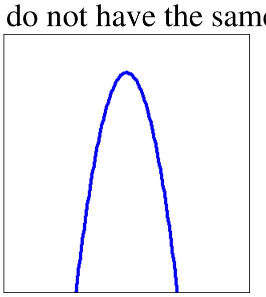
Example A-2: Given $f(x) = 2x - 3$, find the *average rate of change* over the interval [x, $x + h$].

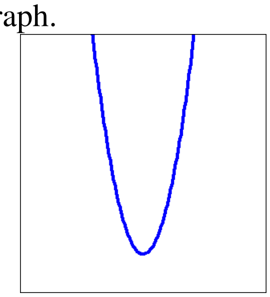


Back in Algebra 1, the slope for a line was defined as a rate of change: $m = \frac{\Delta y}{\Delta x} = \frac{\text{change in } y}{\text{change in } x}$.

However, the slope of a line [$y = mx + b$, slope = m] is always the same.
That is, linear functions have a constant rate of change.

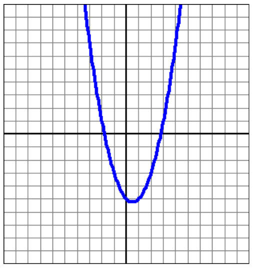
Functions that are not linear do not have a constant rate of change.
That is, their graphs do not have the same slope at every point on the graph.



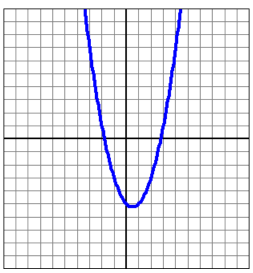


For example, the parabolic shape of a quadratic function will change (at the vertex) from increasing ("going up") to decreasing ("going down"), or vice versa.

Example B-1: Given $f(x) = x^2 - x - 5$, find the *average rate of change* over the interval [– 1, 3].



Example B-2: Given $f(x) = x^2 - x - 5$, find the *average rate of change* over the interval [x, $x + h$].



Note two things: The "average rate of change" is the same as the slope of the secant line connecting the two points. Also, we have merely calculated the difference quotient, $\frac{f(x_2) - f(x_1)}{x_2 - x_1}$, $\frac{f(x+h) - f(x)}{h}$.

Does average rate of change have any practical applications?
In cases where we have some information but not the equation, the average rate of change of two points very close together can give us a good estimate of the instantaneous rate of change, and vice versa.

Example C: When you start a job at a local restaurant, you earn $5.75 per hour. A year later you get a raise to $6.60 per hour. A year and a half after the first raise you get another, to $7.70 per hour. In which space of time did your earnings have a higher rate of change?

The average rate of change provides a way to compare raises given over differing periods of time.