Math 115 Chapter 2.7 Examples and Extra Notes

Arithmetic combination (add, subtract, multiply, divide)

Definitions:
$$(f+g)(x)=f(x)+g(x) \qquad (f-g)(x)=f(x)-g(x)$$
$$(fg)(x)\text{ or }(f \bullet g)(x)=f(x)\bullet g(x) \qquad \left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=f(x)\div g(x)$$

For the examples below, $f(x)=2x+3$ and $g(x)=\sqrt{x}-1$.

When combining functions and evaluating at a specific value of x, you can choose (A) to combine first to get the algebraic rule, then evaluate, or you can choose (B) to evaluate the functions first, then combine them. Illustrations of both approaches are provided in side-by-side columns below.

(A) Find algebraic rule, then evaluate.	**(B) Evaluate first, then combine.**
$(f+g)(x)=(2x+3)+\left(\sqrt{x}-1\right)$ $\quad=2x+\sqrt{x}+2$ $(f+g)(4)=2(4)+\sqrt{4}+2=8+2+2$ $\quad=12$	$f(4)=2(4)+3=11$ $g(4)=\sqrt{4}-1=2-1=1$ $(f+g)(4)=f(4)+g(4)$ $\quad=11+1=12$
$(f-g)(x)=(2x+3)-\left(\sqrt{x}-1\right)$ $\quad=2x+3-\sqrt{x}+1$ $\quad=2x-\sqrt{x}+4$ $(f-g)(4)=2(4)-\sqrt{4}+4=8-2+4=10$ **Note: Don't forget to distribute the negative.**	$f(4)=2(4)+3=11$ $g(4)=\sqrt{4}-1=2-1=1$ $(f-g)(4)=f(4)-g(4)$ $\quad=11-1$ $\quad=10$
$(f \bullet g)(x)=(2x+3)\left(\sqrt{x}-1\right)$ $\quad=2x\sqrt{x}-2x+3\sqrt{x}-3$ $\quad=-2x+2x\sqrt{x}+3\sqrt{x}-3$ $(f \bullet g)(4)=-2(4)+2(4)\left(\sqrt{4}\right)+3\left(\sqrt{4}\right)-3$ $\quad=-8+16+6-3=11$	$f(4)=2(4)+3=11$ $g(4)=\sqrt{4}-1=2-1=1$ $(f \bullet g)(4)=f(4)\bullet g(4)$ $\quad=11\bullet 1$ $\quad=11$
$\left(\dfrac{f}{g}\right)(x)=\dfrac{2x+3}{\sqrt{x}-1}$ $\left(\dfrac{f}{g}\right)(4)=\dfrac{2(4)+3}{\sqrt{4}-1}=\dfrac{8+3}{2-1}=\dfrac{11}{1}$ $\quad=11$	$f(4)=2(4)+3=11$ $g(4)=\sqrt{4}-1=2-1=1$ $\left(\dfrac{f}{g}\right)(4)=\dfrac{f(4)}{g(4)}=\dfrac{11}{1}$ $\quad=11$

Domains and ranges of arithmetic combinations
The domain and range of arithmetic combinations of functions may differ from those of the original functions. (See column A above for the algebraic rules.)

Function	Domain	Range
f	$(-\infty, \infty)$	$(-\infty, \infty)$
g	$[0, \infty)$	$[-1, \infty)$
$f+g$	$[0, \infty)$	$[2, \infty)$
$f-g$	$[0, \infty)$	$[4, \infty)$
fg	$[0, \infty)$	$[-3, \infty)$
$\dfrac{f}{g}$	$[0, 1)\cup(1, \infty)$	$(-\infty, 3]\cup(0, \infty)$

Composition of functions

Another way to combine functions is composition: evaluating one function "within" the other. Composition can be written either as "$(f \circ g)(x)$" or "$f \circ g$", read as "f composed with g" or "f of $g(x)$". For the examples below, $f(x) = 2x + 3$, $g(x) = \sqrt{x} - 1$, and $h(x) = \sqrt{x-1}$. (Note that g and h are not the same!)

$$(f \circ g)(x) = f(g[x])$$
$$= f\left(\sqrt{x} - 1\right)$$
$$= 2\left(\sqrt{x} - 1\right) + 3$$
$$= 2\sqrt{x} - 2 + 3$$
$$= 2\sqrt{x} + 1$$

$$(g \circ f)(x) = g(f[x])$$
$$= g(2x + 3)$$
$$= \sqrt{2x + 3} - 1$$

$$(h \circ f)(x) = h(f[x])$$
$$= h(2x + 3)$$
$$= \sqrt{(2x + 3) - 1}$$
$$= \sqrt{2x + 2}$$

Domains and ranges of compositions

The domain and range of compositions of functions may differ from those of the original functions.

Function	Domain	Range
f	$(-\infty, \infty)$	$(-\infty, \infty)$
g	$[0, \infty)$	$[-1, \infty)$
$f \circ g$	$[0, \infty)$	$[1, \infty)$
$g \circ f$	$\left[-\dfrac{3}{2}, \infty\right)$	$[-1, \infty)$

Evaluating compositions at specific points

When doing function composition and evaluating at a specific value of x, the same principle applies that was illustrated above for arithmetic combinations. You can choose (A) to do the composition first to get the algebraic rule, then evaluate, or you can choose (B) to evaluate the functions first, then do the composition.

(A) Find algebraic rule, then evaluate.	**(B) Evaluate first, then combine.**
$(f \circ g)(x) = f\left(\sqrt{x} - 1\right)$ $\quad = 2\left(\sqrt{x} - 1\right) + 3$ $\quad = 2\sqrt{x} - 2 + 3$ $\quad = 2\sqrt{x} + 1$ $(f \circ g)(4) = 2\sqrt{4} + 1 = 2(2) + 1 = 5$	$g(4) = \sqrt{4} - 1 = 2 - 1 = 1$ $(f \circ g)(4) = f(g(4)) = f(1)$ $\quad = 2(1) + 3$ $\quad = 5$
$(g \circ f)(x) = g(2x + 3)$ $\quad = \sqrt{2x + 3} - 1$ $(g \circ f)(4) = \sqrt{2(4) + 3} - 1 = \sqrt{11} - 1$	$f(4) = 2(4) + 3 = 11$ $(g \circ f)(4) = g(f(4))$ $\quad = g(11) = \sqrt{11} - 1$

Using a graphing calculator

You can use a TI-82 or TI-83 to graph and evaluate arithmetic combinations of functions as well as compositions of functions. For these examples $f(x) = 2x + 3$ will be entered as Y1 and $g(x) = \sqrt{x} - 1$ will be entered as Y2. The combinations will be entered in terms of Y1 and Y2 using the calculator's Y-VARS function.

Combination	Calculator entry	Combination	Calculator entry
$f + g$	Y3 = Y1 + Y2	$f - g$	Y3 = Y1 – Y2
fg also written $f \bullet g$	Y3 = Y1 * Y2	f/g	Y3 = Y1/Y2
$f \circ g$	Y3 = Y1 (Y2)	$g \circ f$	Y3 = Y2 (Y1)

Note that a graphing calculator does a very poor job of showing the graph of f/g. You'd need a more sophisticated computer software program.