Precalculus 115, section 3.2 Polynomial Functions

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Definition: A **polynomial term** has a number coefficient, a variable, and an integer exponent greater than or equal to 0 on the variable.

term	coefficient	variable	exponent	polynomial term?
$5x^4$				
$-6x^3$				
x^7				
$\sqrt{3x}$				
$\sqrt{3}\,x$				
4				
$\frac{4}{x}$				

The **degree** of a polynomial term is the value of its exponent.

The **degree** of a polynomial function is the value of the highest degree polynomial term.

What is the domain of every polynomial function?

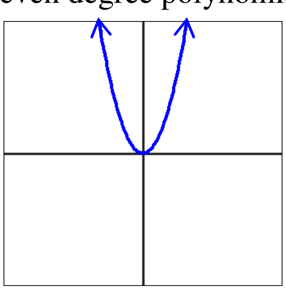
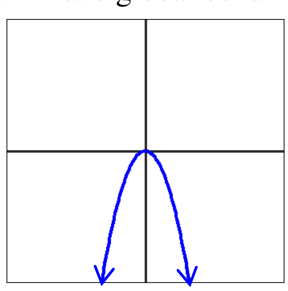
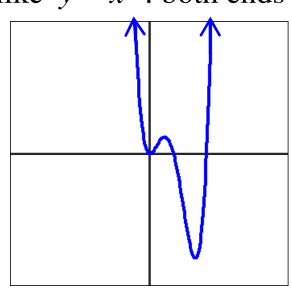
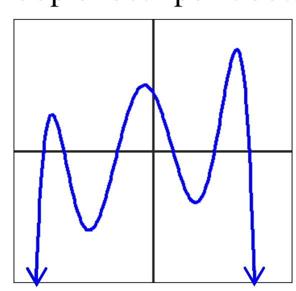
When considering the graph of a polynomial function, we'll first look at two things:

- global behavior, i.e. whether the ends point up or down
- local behavior at the *x*-intercepts, i.e. the shape of the graph at those intercepts.

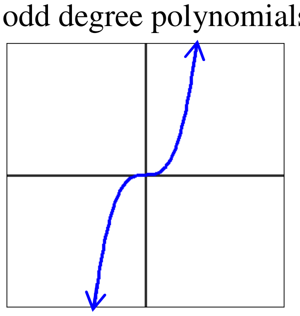
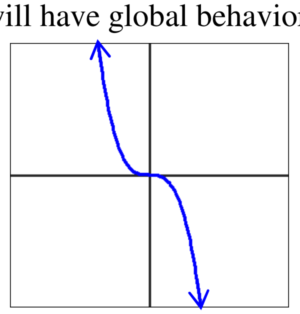
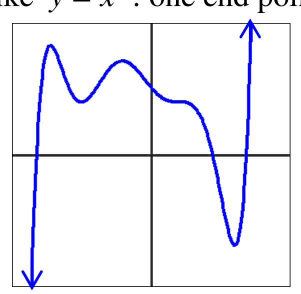
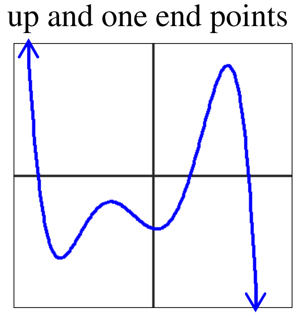
Once those are determined, we'll use a variation of a table of signs to fill in the rest.

For global (end) behavior, all you really need to know is the global behavior of two very basic polynomials.

All even degree polynomials will have global behavior like $y = x^2$: both ends point up or both point down.

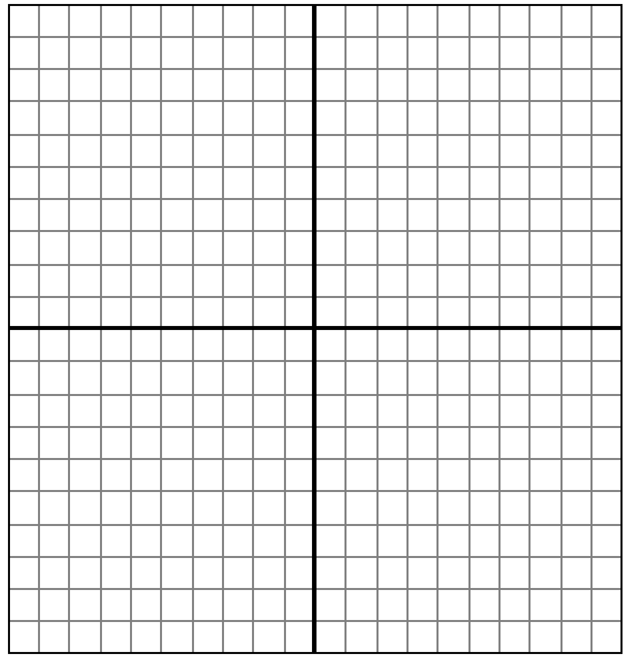





All odd degree polynomials will have global behavior like $y = x^3$: one end points up and one end points down.

We'll explore local behavior at the *x*-intercepts in the Examples below.

Example A: Sketch the graph of $f(x) = (x-3)(x-1)(x+2)$.



degree:

global (end) behavior:

y-intercept:

factors and multiplicity:

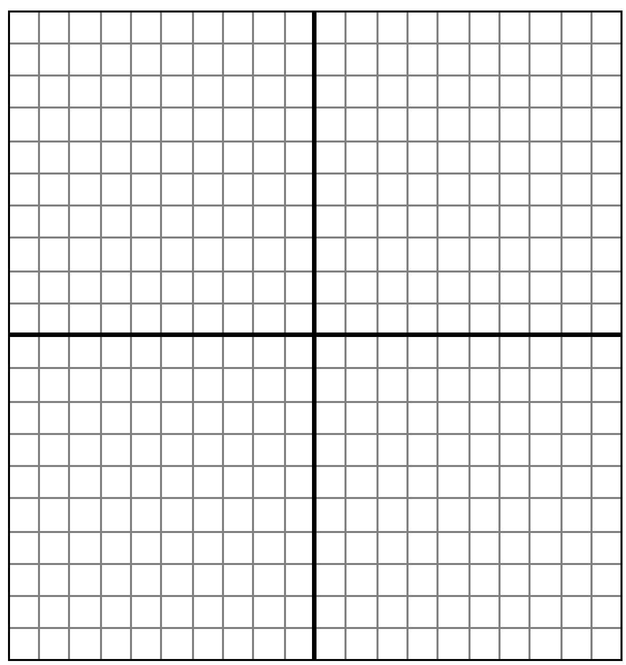
zeroes:

x-intercepts:

local behavior at each x-intercept:

For this class, we won't worry about how high or low the local maxima and minima should be. (That determination will come with Calculus.) For quizzes and exams, you'll need to have the correct global behavior, correct local behavior at x-intercepts, correct placement above or below the x-axis, and correct y-intercept.

Example B: Sketch the graph of $f(x) = (x-3)(x-1)^2(x+2)$.



degree:

global (end) behavior:

y-intercept:

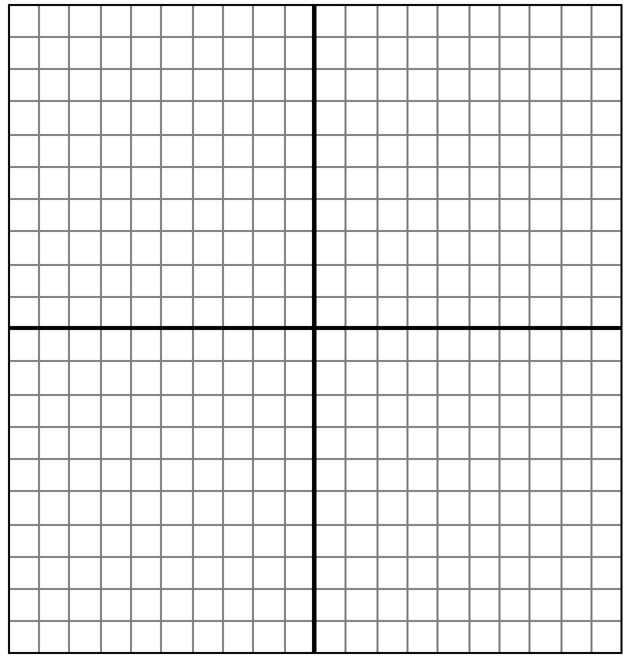
factors and multiplicity:

zeroes:

x-intercepts:

local behavior at each x-intercept:

Example C: Sketch the graph of $f(x) = -(x-3)(x-1)^3(x+2)$.



degree:

global (end) behavior:

y-intercept:

factors and multiplicity:

zeroes:

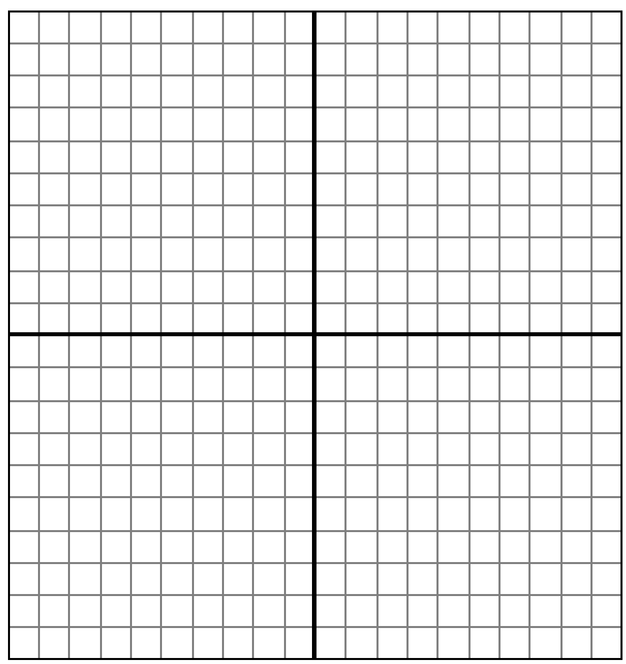
x-intercepts:

local behavior at each x-intercept:

Based on the work we've done so far, what is the maximum number of zeroes (i.e. x-intercepts) an n^{th} degree polynomial function can have?

What is the maximum number of "turning points (i.e. local extrema)?

Example D: Sketch the graph of $f(x) = x^3 - x^2 - 9x - 9$.



degree:

global (end) behavior:

y-intercept:

factors and multiplicity:

zeroes:

x-intercepts:

local behavior at each x-intercept:

Example E: Find a function that could have the graph pictured below.