# Precalculus 115, sections 4.1 Exponential Functions

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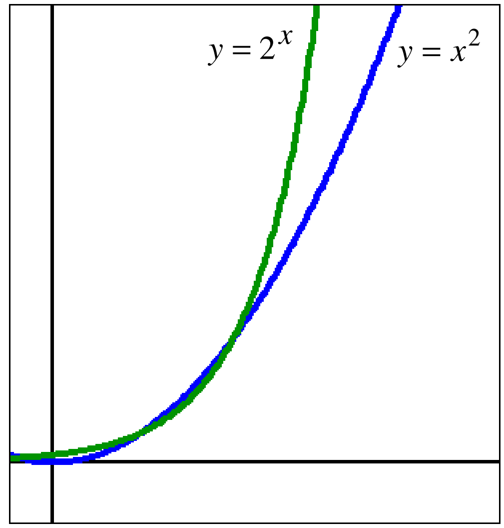
Take a piece of paper, and fold it in half. You've doubled the number of layers—from 1 to 2. Fold it in half again, and you've once again doubled the layers—from 2 to 4. After the next fold you'd have 8 layers. After the next, 16. Then 32. Then 64, 128, 256, 512, 1024, etc. After just 10 foldings, we have the paperback edition of just the first volume of *Lord of the Rings* that we're attempting to fold in half.

When $x$ is the base, we have a *power function*. When $x$ is the exponent we have an *exponential function*. The scenario above illustrates the exponential function $y = 2^x$.



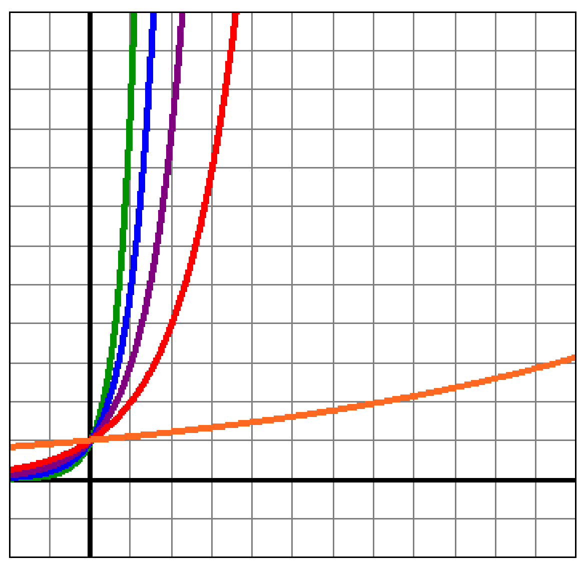

If we compare the graph of $y = x^2$ to the graph of $y = 2^x$, we can see that for positive values of $x$ the exponential function grows much more quickly than the power function. (Thus our difficulty in folding a piece of paper in half successive times.)

Exponential functions have many applications because they model many kinds of growth and shrinking: e.g. populations, bank deposits, radioactive decay.

All of the usual properties apply to exponential functions:

$$b^x * b^y = b^{x+y} \qquad \frac{b^x}{b^y} = b^{x-y} \qquad \frac{1}{b^y} = b^{0-y} = b^{-y} \qquad \left(b^x\right)^y = b^{xy} \qquad a^x * b^x = (ab)^x \qquad \frac{a^x}{b^x} = \left(\frac{a}{b}\right)^x.$$

Functions with the basic form $y = b^x$ are actually a family of functions. We'll consider only values for $b$ that are positive. (Negative values of $b$ are extremely problematic, since even and odd values of $x$ would cause $y$ to fluctuate between positive and negative.)



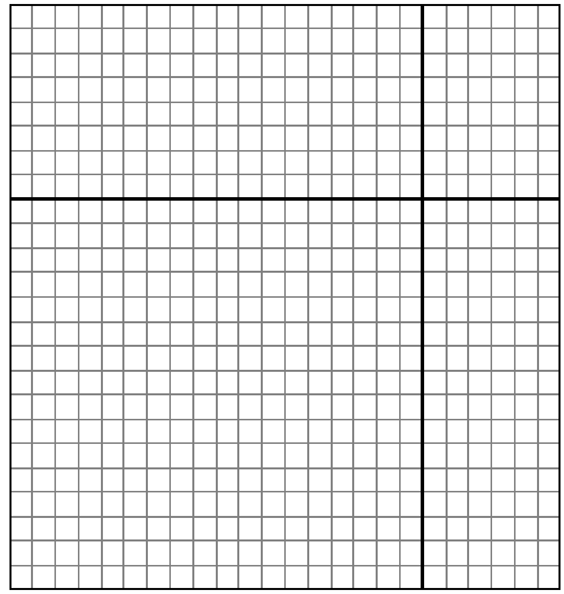
Consider the functions $y = 10^x$, $y = 5^x$, $y = 3^x$, $y = 2^x$, and $y = 1.1^x$ pictured in the graph to the right. The graphs also have similar shape—the major difference is slope/steepness of the curve at specific values of $x$. Note that for $x > 0$ slope of $y = 10^x$ is steepest; slope of $y = 1.1^x$ is most shallow.

Since $b^0 = 1$ for all values of $b \neq 0$, so (0, 1) makes a good reference point, no matter what number we have as the base of our exponential function.

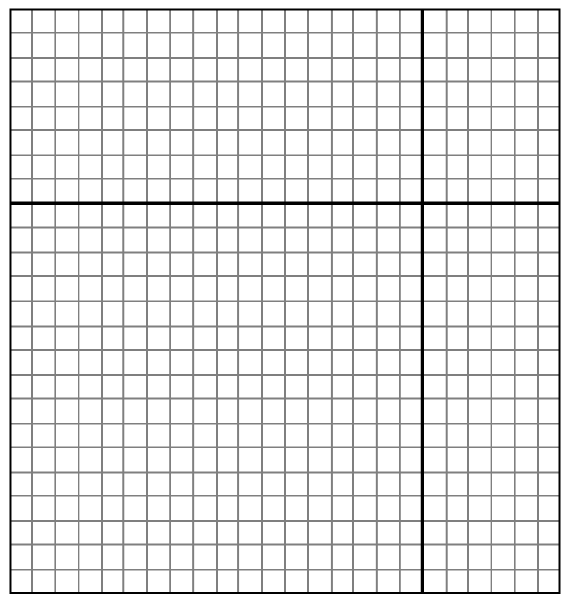
Note also that each of the basic exponential functions has a horizontal asymptote $y = 0$.

When we're graphing exponential functions, we'll start by shifting the reference point and horizontal asymptote. On the way, we'll find $y$- and $x$-intercepts where they exist.

Example A: Sketch the graph of $f(x) = 2^{x+3} - 4$.



Example B: Sketch the graph of $f(x) = -2^{x+3} - 4$.



Example C: Find the exponential function $f(x) = a^{x}$ whose graph is given below.