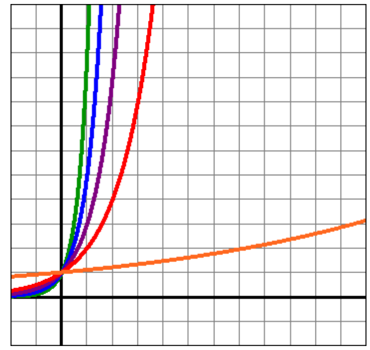
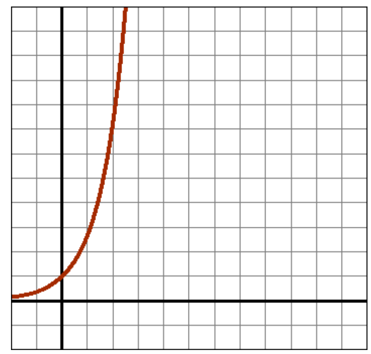
Precalculus 115, section 4.2 Exponential Function e^x
section 4.3a Introduction to Logarithms

notes by Tim Pilachowski





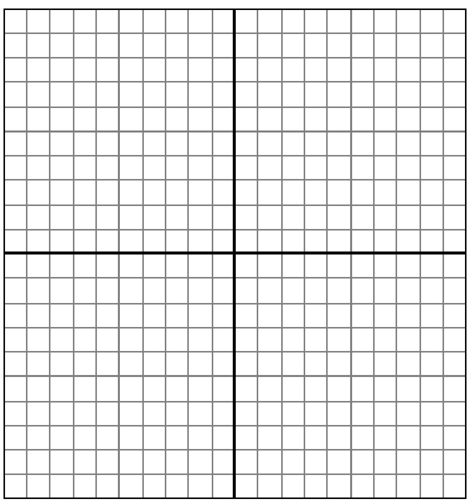
Recall now a point made in Lecture 4.1, exponential functions are a family, with the same basic shape, but different levels of steepness. At $x = 0$, the graph of $y = 10^x$ is the steepest; the graph of $y = 1.1^x$ is most shallow. Somewhere in that family must be a base for which the slope of a line tangent to the curve at the point (0, 1) is exactly 1. This number is *e*, Euler's number. Like π or $\sqrt{2}$, *e* is an irrational number. The value of *e* is *approximately* 2.7. (One decimal place will be sufficient for our purposes.) The corresponding function, $y = e^x$, is called the natural exponential function.

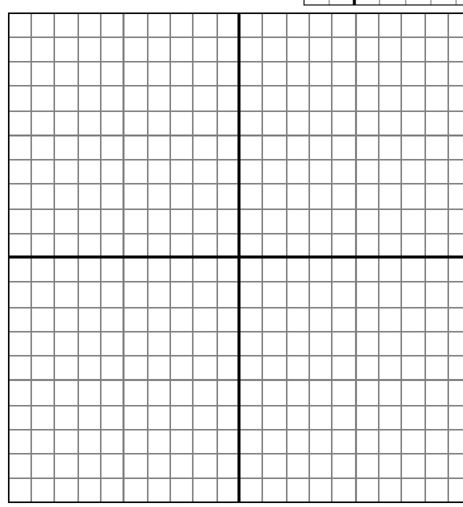
The "definition" of the number *e* given above is a non-technical one used in Math 220: Calculus with Applications. More technical definitions would be

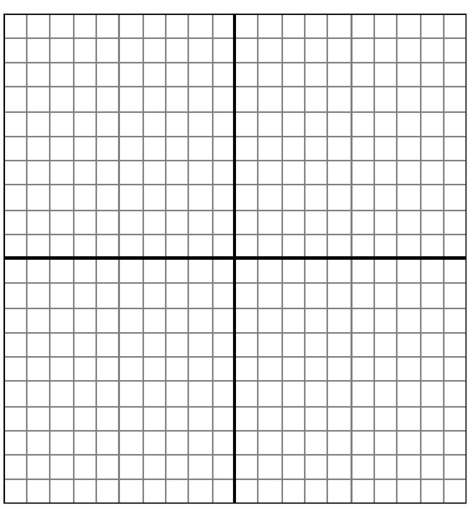
$$e = \lim_{n \to \infty}\left(1 + \frac{1}{n}\right)^n \quad \text{and} \quad e = \sum_{n=0}^{\infty} \frac{1}{n!}.$$

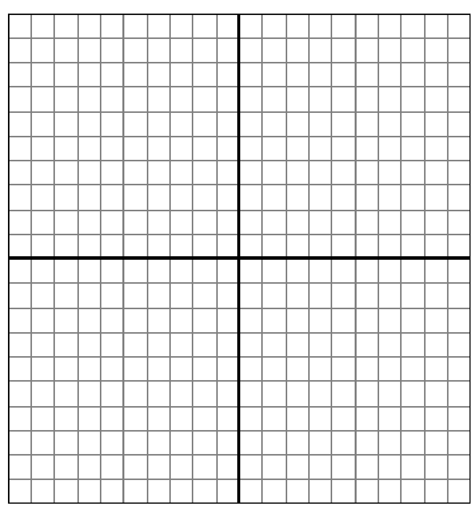
You'll run across both of these in Math 141.

Example A: Compare the graphs of $y = e^x$, $y = -e^x$, $y = e^{-x}$ and $y = -e^{-x}$.

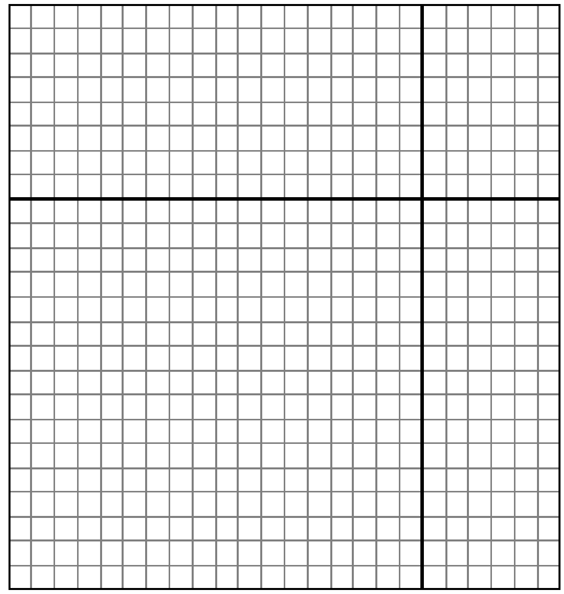








Example B: Sketch the graph of $f(x) = -e^{x-3} + 1$.



section 4.3a

Definition: A **logarithm** function is the inverse to an exponential function. Specifically,

$$\log_b(x) = n \quad \Leftrightarrow \quad b^n = x.$$

logarithmic form exponential form

Because logarithm functions and exponential functions are inverses of each other, they exhibit a property which will be very useful to us in evaluating, simplifying, and solving. Given $f(x) = \log_b(x)$ and $g(x) = b^x$, it will always be true that $f \circ g = \log_b(b^x) = x$ and $g \circ f = b^{\log_b(x)} = x$.

Examples C. Evaluate each of the following logarithms.

1. $\log_2(4)$
2. $\log_2\left(\frac{1}{8}\right)$
3. $\log_{\frac{1}{2}}(8)$
4. $\log_2(1)$
5. $\log_4(2)$
6. $\log_4(32)$
7. $\log_4\left(\sqrt{2}\right)$
8. $\log_{\sqrt{2}}(16)$

Examples D. Evaluate each of the following logarithms.

1. $\log(100)$
2. $\log(0.0001)$
3. $\ln\left(e^5\right)$
4. $\ln\left(\sqrt[3]{e^7}\right)$