Precalculus 115, section 4.3b Logarithm Functions

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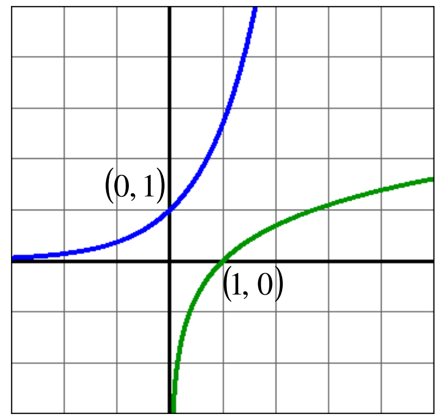
Definition: A **logarithm** function is the inverse to an exponential function. Specifically,

$$\log_b(x) = y \quad \Leftrightarrow \quad b^y = x.$$

logarithmic form exponential form

Because logarithm functions and exponential functions are inverses of each other, they exhibit a property which will be very useful to us in evaluating, simplifying, and solving. Given $f(x) = \log_b(x)$ and $g(x) = b^x$, it will always be true that $f \circ g = \log_b(b^x) = x$ and $g \circ f = b^{\log_b(x)} = x$.

Functions with the basic form $y = \log_b(x)$ are actually a family of functions. We'll consider only values for b that are greater than 1.



Another characteristic of inverse functions is that the graph of f and the graph of f^{-1} are symmetric with respect to the line $y = x$. Graphs of a generic exponential function and its inverse logarithm function are pictured to the right.

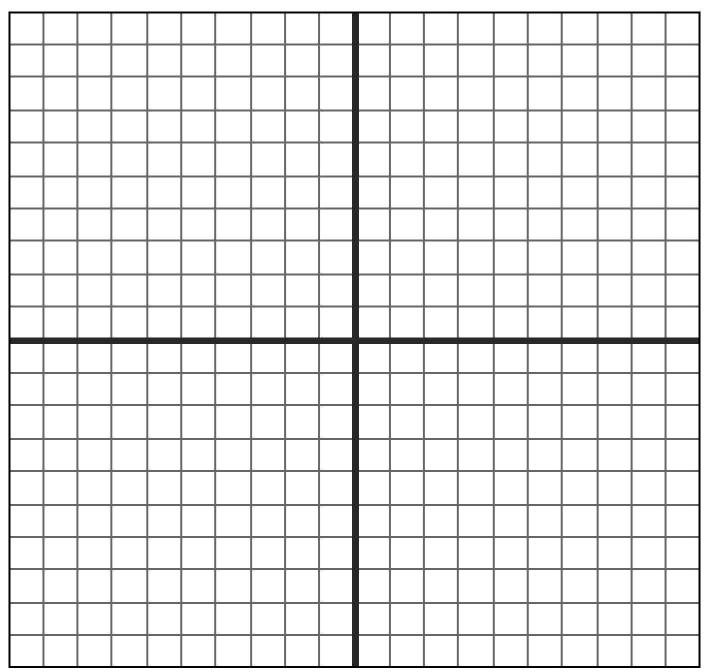
Since $b^0 = 1$, it necessarily follows that $\log_b(1) = 0$ for all bases b. The point (1, 0) will be our reference point for logarithm graphs, no matter what number we have as the base of our logarithm function.

Note also that each of the basic logarithm functions has a vertical asymptote $x = 0$.

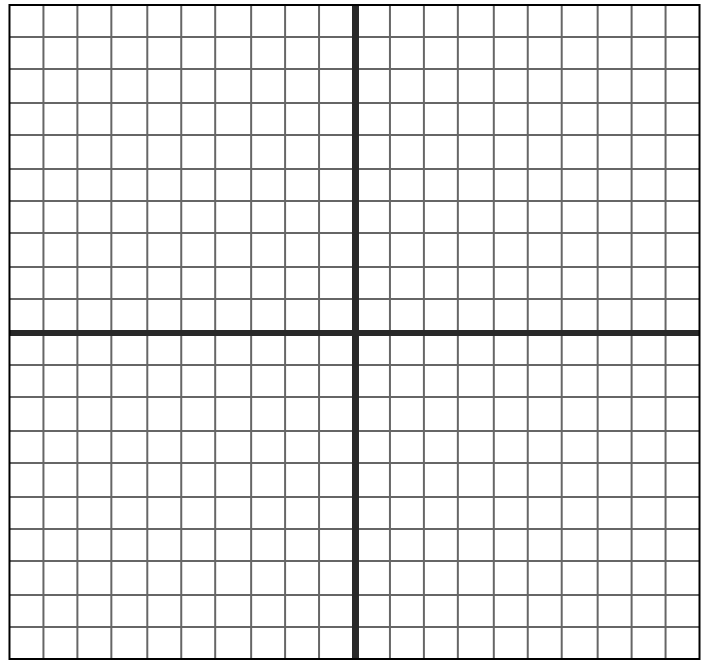
The domain of a basic logarithm function is $(0, \infty)$, and the range of a basic logarithm function is $(-\infty, \infty)$.

When we're graphing logarithm functions, we'll start by shifting the reference point and vertical asymptote. On the way, we'll find y- and x-intercepts where they exist.

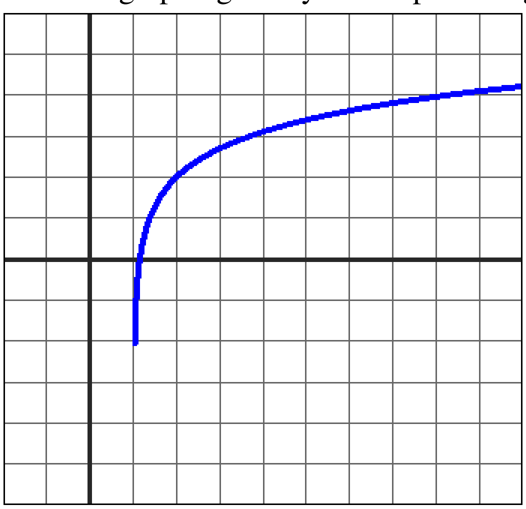
Example A: Sketch the graph of $f(x) = \log_2(x+3) - 1$.

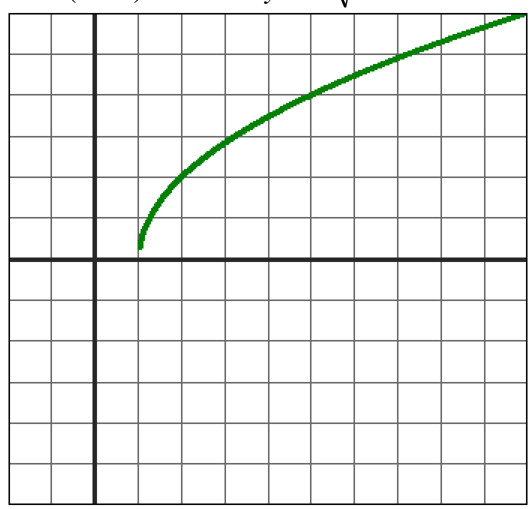


Example B: Sketch the graph of $f(x) = -\ln(3-x) - 1$.



Example C: Use a graphing utility to compare the graphs of $y = \ln(x-1) + 2$ and $y = 2\sqrt{x-1}$.





Example D: Find the logarithm function $f(x) = \log_b x$ whose graph is given below.