Precalculus 115, section 5.2 Basic Trigonometric Functions

notes by Tim Pilachowski

We now come to the basic trigonometric functions. The first two, sine and cosine, we'll define in terms of terminal points on the unit circle.

Given an angle t measured in radians and placed in standard position on a unit circle, the cosine of t is defined to be the x-coordinate of the terminal point. The sine of t is defined to be the y-coordinate of the terminal point. The functions are written $f(t) = \cos t$ and $f(t) = \sin t$ respectively.

What is the domain of $f(t) = \cos t$? of $f(t) = \sin t$?

What is the range of $f(t) = \cos t$? of $f(t) = \sin t$?

To evaluate cosine and sine for specific angles, we'll first identify the appropriate reference angle in Quadrant I, then use the symmetry of the unit circle to determine whether the sine and cosine are positive or negative.

Example A: Find the cosine and sine of angles $\pi, -\frac{7\pi}{2}, \frac{\pi}{6}, -\frac{4\pi}{3}$ and $\frac{13\pi}{4}$.

The other four basic trigonometric functions (cosecant, secant, tangent, cotangent) will be defined in terms of sin t and cos t.

$$\sec t = \frac{1}{\cos t} \qquad \csc t = \frac{1}{\sin t} \qquad \tan t = \frac{\sin t}{\cos t} \qquad \cot t = \frac{\cos t}{\sin t} = \frac{1}{\tan t}$$

Example A continued: Find the secant, cosecant, tangent and cotangent of angles $\pi, -\frac{7\pi}{2}$ and $-\frac{4\pi}{3}$.

Recall from Lecture 5.1, the equation of the unit circle is $x^2 + y^2 = 1$. When we replace the x-coordinate and y-coordinate with the appropriate trigonometric functions, we get the trig version of the Pythagorean identity.

$$x^2 + y^2 = (\cos t)^2 + (\sin t)^2 = \cos^2 t + \sin^2 t = 1$$

Memorize this identity. Your text has two other versions. I'd suggest deriving them as needed.

$$\frac{\cos^2 t}{\cos^2 t} + \frac{\sin^2 t}{\cos^2 t} = \frac{1}{\cos^2 t} \quad \Rightarrow \quad 1 + \tan^2 t = \sec^2 t \qquad\qquad \frac{\cos^2 t}{\sin^2 t} + \frac{\sin^2 t}{\sin^2 t} = \frac{1}{\sin^2 t} \quad \Rightarrow \quad \cot^2 t + 1 = \csc^2 t$$

Example B: Given sin $t = 0.3$ and t in Quadrant II, find the values of the other five trigonometric functions.

Example C: Given $\tan t = -\frac{3}{2}$ and t in Quadrant II, find the values of the other five trigonometric functions.

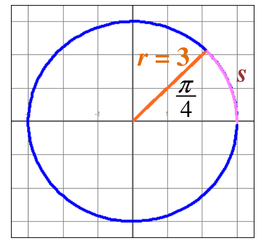
Example D: Write cos t in terms of sin t for t in Quadrant IV.

Example E: Write cot t in terms of sin t for t in Quadrant IV.

Side topic for another view of radian measure: circles other than the unit circle. (You do not have to know this. I include it only for getting a little more experience thinking in terms of radians.)

The circumference formula, $C = 2\pi r$, is linear, and allows us to state that the arc lengths of two different circles are proportional. We can use this fact to find arc lengths in circles with radii not equal to 1.

Example F: In a circle with radius 3, what is the arc length s cut off by a central angle of $\frac{\pi}{4}$ radians?



By definition, $\frac{\pi}{4}$ radians takes us around the unit circle for an arc length of $\frac{\pi}{4}$.

By the proportionality noted above, a circle of radius 3 will have arc lengths 3 times those of the unit circle, so $\frac{\pi}{4}$ radians takes us around the larger circle for an arc length of $\frac{3\pi}{4}$.

Note that this is supported by what we know of circles.

When the radius = 3, the circumference = 6π. A central angle of $\frac{\pi}{4}$ (or 45º) takes us around $\frac{1}{8}$ of the circumference, and $\frac{1}{8} * 6\pi = \frac{3\pi}{4}$.

In general, we can state, Arc Length = Radians times Radius.

Example G: If you are standing on the equator, as the Earth rotates you are actually moving through three-dimensional space, even though it doesn't feel that way. How far do you move in 1 hour? (The radius of Earth at the equator is approximately 4000 miles.)

In the course of a day the Earth rotates once around, i.e. 2π radians.

One hour is $\frac{1}{24}$ of a day, and so you move $\frac{2\pi}{24} = \frac{\pi}{12}$ radians.

Since the Earth's radius is approximately 4000 miles, the distance traveled is

$$\frac{\pi}{12} * 4000 = \frac{1000\pi}{3}$$ miles (*exact* answer).

This is *approximately* 1047 miles.

We'll need this idea of proportionality of circles later on, in Examples, group work, and homework exercises.