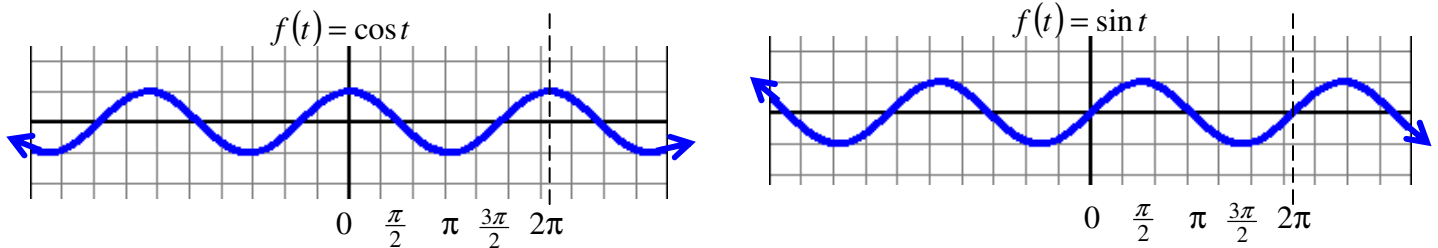# Precalculus 115, section 5.3 Graphs of Trigonometric Functions

notes by Tim Pilachowski

Both $f(t) = \cos t$ and $f(t) = \sin t$ are *periodic functions*. The graphs are pictured below.

The domain of both functions is all $(-\infty, \infty)$, since we can go around the unit circle in either direction as many times as we want. The range of each is $[-1, 1]$.

Applications which sometimes use sine and cosine to model periodic behavior include: temperature fluctuations, seasonal sales, regular breathing, populations of migratory animals, and electric currents.

For graphing sine and cosine functions, while we'll still be using shifts and translations, the periodic nature of trigonometric functions means that we'll need some new terminology: amplitude, period, and phase shift.
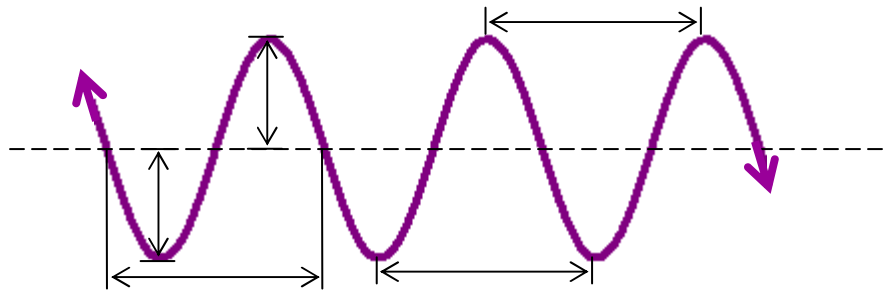
The **amplitude** of a sine or cosine function is the vertical distance the graphs go up and down from the middle. The **period** is of a sine or cosine function is the horizontal distance the graph needs to complete one cycle.

Note that each period begins and ends with the same kind of point (intercept, minimum, maximum).
Each minimum is exactly halfway between two maxima.
Each maximum is exactly halfway between two minima.
Each extreme (max or min) is exactly halfway between two intercepts.
Each intercept is exactly halfway between a maximum and a minimum.

Both $f(t) = \cos t$ and $f(t) = \sin t$ have an amplitude of 1. (The graphs go up 1 and down 1 from the middle.)
Both $f(t) = \cos t$ and $f(t) = \sin t$ have a period equal to 2π. (Horizontal scale below is $\pi/2$.)
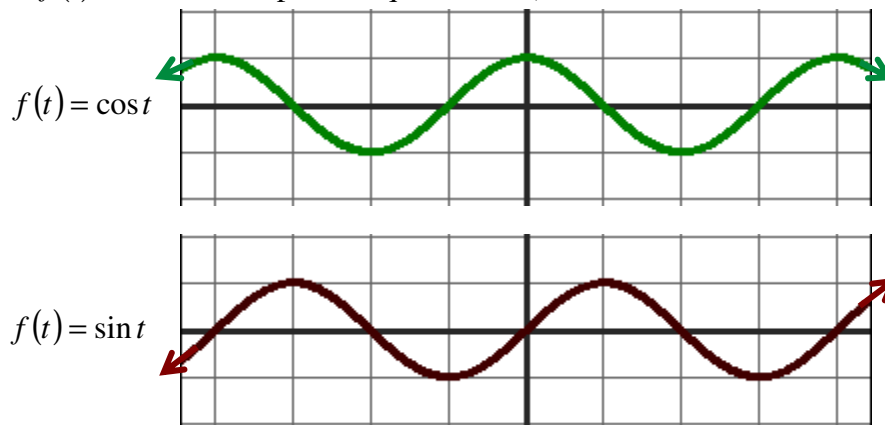
Note also that both $f(t) = \cos t$ and $f(t) = \sin t$ have the same shape, shifted horizontally by $\frac{\pi}{2}$.

While the *vertical* shift of trigonometric functions will be the same as it was for the functions in section 2.5, the *horizontal* shift is a bit trickier now, due to the periodic nature of trigonometric functions.

2.5 Example F revisited: Compare the graphs of $f(x) = \sin x$, $p(x) = \sin(3x)$ and $q(x) = \sin\left(\frac{1}{3}x\right)$.
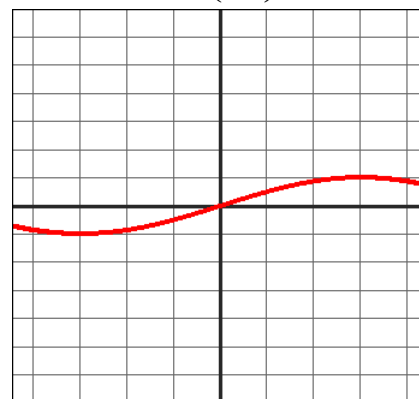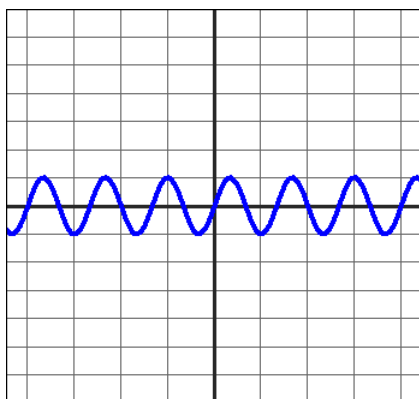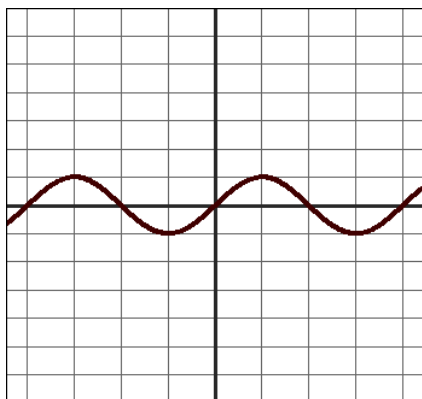
We'll have to take the change of period into account along with the horizontal shift. The horizontal shift will get a new name, **phase shift**, and in practice we'll apply it to the y-intercept of a trigonometric graph.

The standard forms of the cosine and sine functions are
$$y = a\cos[k(x-b)] + c \quad \text{and} \quad y = a\sin[k(x-b)] + c.$$
amplitude $= |a|$, period $= \dfrac{2\pi}{|k|}$, phase shift $= b$ [applied to y-intercept of the basic function], vertical shift $= c$
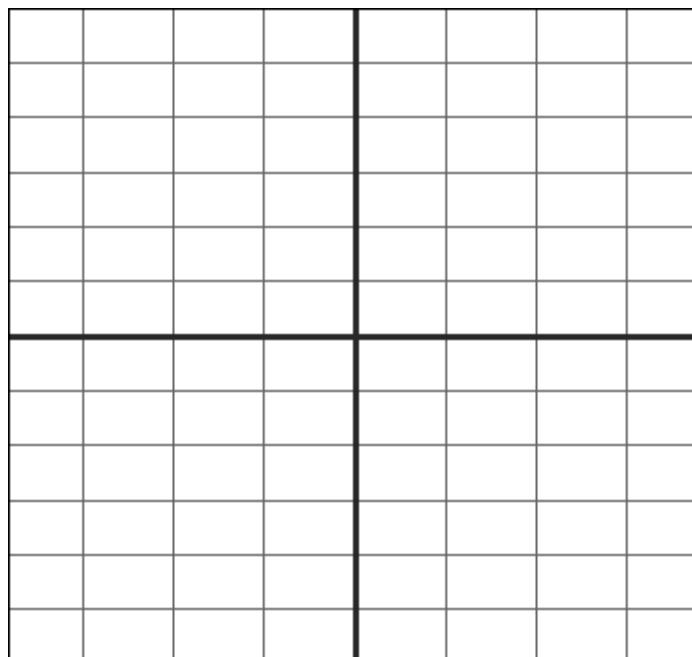
Example A: Sketch the graph of $f(x) = 1 - \sin x$.

amplitude:

period:

phase shift:

points on the graph and their nature:
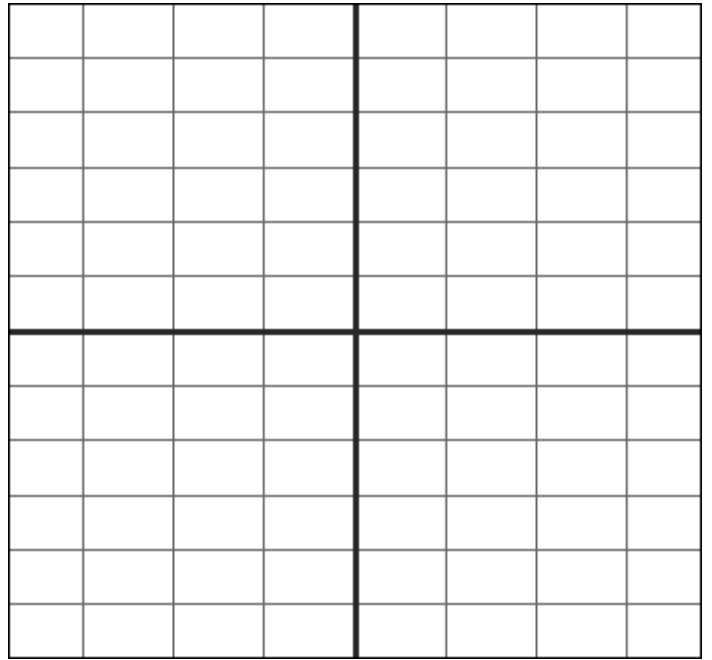

y-intercept:

Example B: Sketch the graph of $f(x) = \cos(3x)$.

amplitude:

period:

phase shift:

points on the graph and their nature:
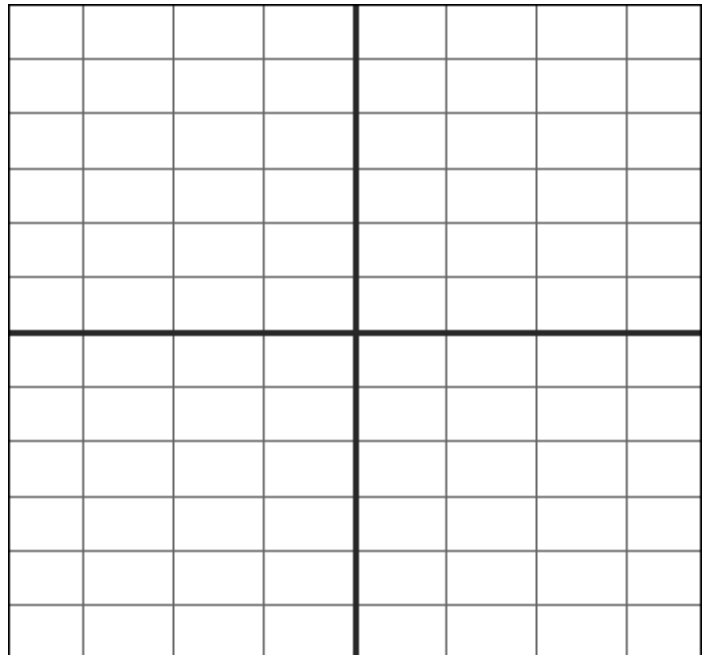
y-intercept:

Example C: Sketch the graph of $f(x) = 3\cos\left(x + \dfrac{\pi}{6}\right)$.

amplitude:

period:

phase shift:

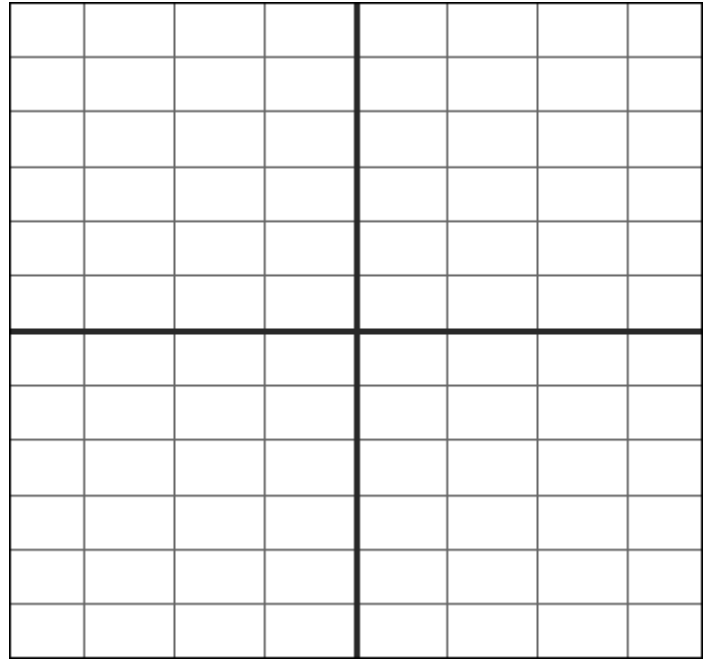points on the graph and their nature:

y-intercept:

Example D: Sketch the graph of $f(x) = \sin\left(3x + \dfrac{\pi}{2}\right)$.

amplitude:

period:

phase shift:

points on the graph and their nature:

y-intercept:

Example E: Identify the period of $g(x) = \sin \pi x$ and $h(x) = \cos\dfrac{\pi}{3}x$.