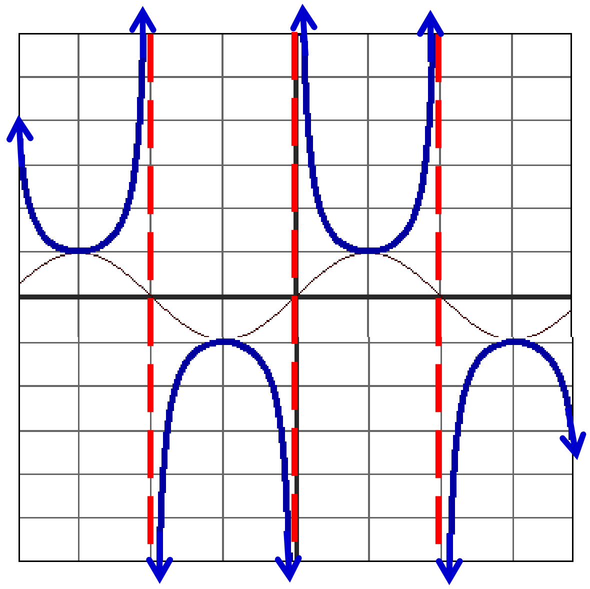
Precalculus 115, section 5.4 More Graphs of Trigonometric Functions

notes by Tim Pilachowski



Since $\csc t = \dfrac{1}{\sin t}$, the graphs of $y = \sin t$ and $y =$ cosecant t are closely related.

Wherever sine t equals 0, the graph of $y = \csc t$ has a vertical asymptote.

As sine t approaches 0, the graph of $y = \csc t$ will go up or down along a vertical asymptote.

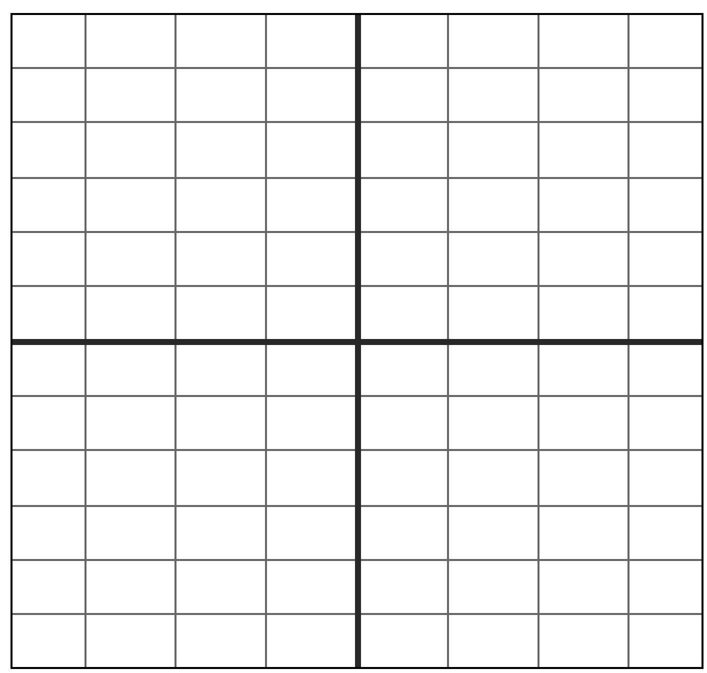
Wherever sine t equals 1, the graph of $y = \csc t$ will also have a
y-coordinate equal to 1. Wherever $y = \sin t$ has a maximum, the graph of $y = \csc t$ will have a minimum.

Wherever sine t equals -1, the graph of $y = \csc t$ will also have a y-coordinate equal to -1. Wherever $y = \sin t$ has a minimum, the graph of $y = \csc t$ will have a maximum.

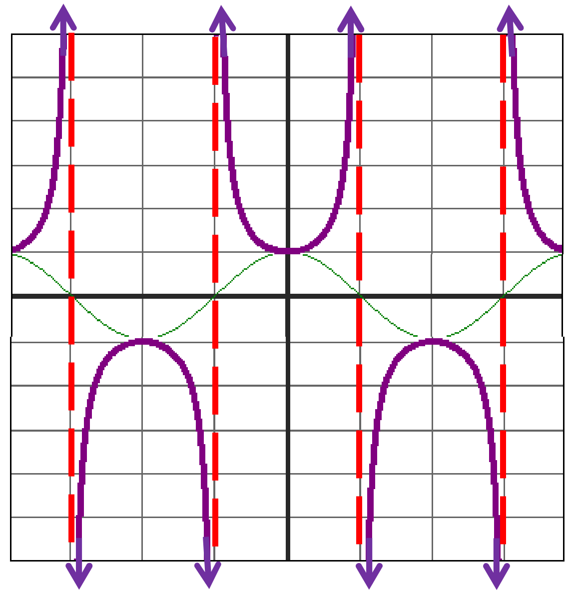
Note that maxima and minima are exactly halfway between asymptotes. Also note that asymptotes are exactly halfway between a maximum and a minimum.

My suggestion is that you draw the associated sine graph first, next insert the necessary vertical asymptotes, then put maxima and minima in place, and finally draw the curve.

Example A: Sketch the graph of $y = \csc\left(3x - \dfrac{\pi}{2}\right)$. (Note that the x-axis is scaled by $\pi/2$.)



Since $\sec t = \dfrac{1}{\cos t}$, the graphs of $y = \cos t$ and $y = \text{secant } t$ are closely related.



Wherever cosine t equals 0, the graph of $y = \sec t$ has a vertical asymptote.

As cosine t approaches 0, the graph of $y = \sec t$ will go up or down along a vertical asymptote.

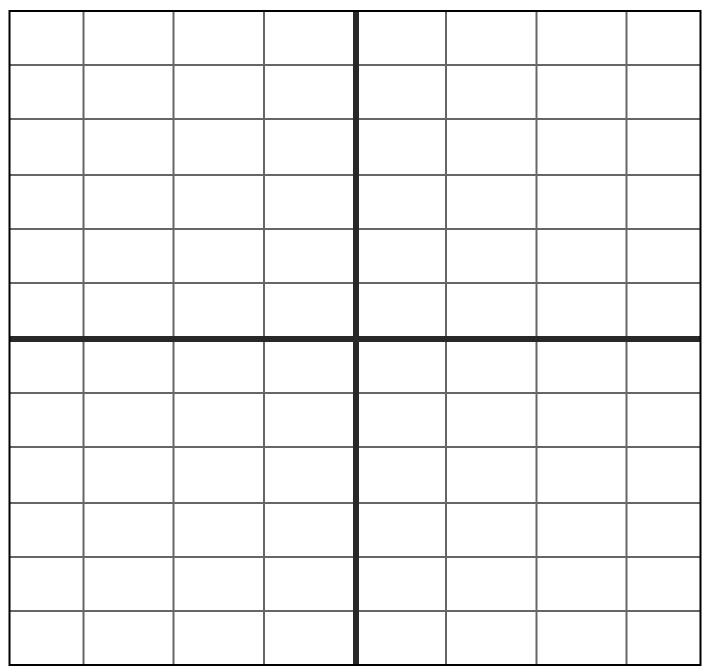
Wherever cosine t equals 1, the graph of $y = \sec t$ will also have a y-coordinate equal to 1. Wherever $y = \cos t$ has a maximum, the graph of $y = \sec t$ will have a minimum.

Wherever cosine t equals -1, the graph of $y = \sec t$ will also have a y-coordinate equal to -1. Wherever $y = \cos t$ has a minimum, the graph of $y = \sec t$ will have a maximum.

Note that maxima and minima are exactly halfway between asymptotes. Also note that asymptotes are exactly halfway between a maximum and a minimum.

My suggestion is that you draw the associated cosine graph first, next insert the necessary vertical asymptotes, then put maxima and minima in place, and finally draw the curve.

Example B: Sketch the graph of $y = 2\sec\left(x - \dfrac{\pi}{2}\right)$. (Note that the x-axis is scaled by $\pi/2$.)



Since $\tan t = \dfrac{\sin t}{\cos t}$, the graphs of $y = \sin t$, $y = \cos t$ and y = tangent t are closely related.

Wherever sine t equals 0, the graph of $y = \tan t$ will also have a y-coordinate equal to 0.

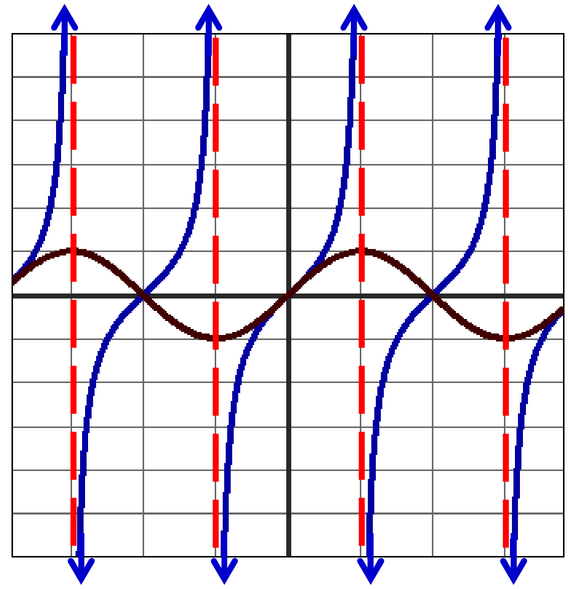
Wherever sine t equals 1, cosine t equals 0, and the graph of $y = \tan t$ has a vertical asymptote.

As cosine t approaches 0, the graph of $y = \tan t$ will go up or down along a vertical asymptote.

The graph of $y = \tan t$ has neither a maximum nor a minimum.

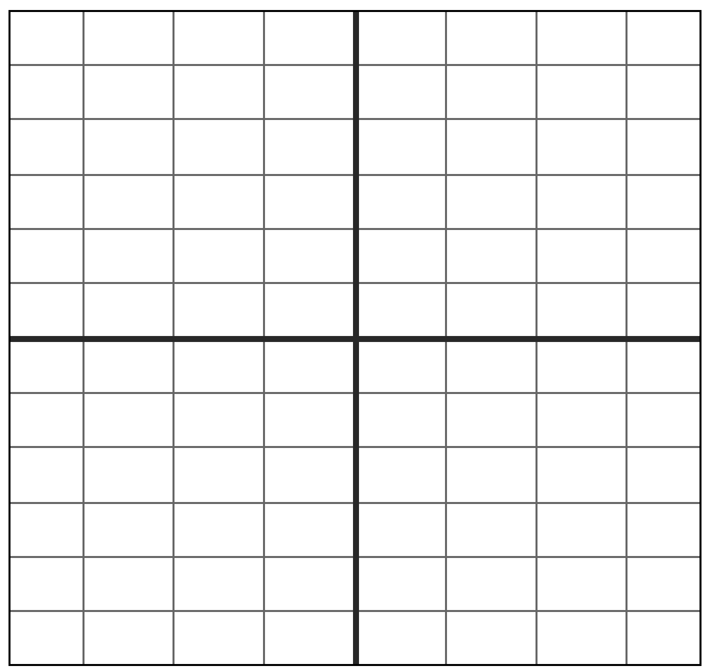
The period of $y = \tan t$ is π.

Note that x-intercepts are exactly halfway between asymptotes. Also note that asymptotes are exactly halfway between x-intercepts.



My suggestion is that you draw the associated sine graph first, next put the x-intercepts in place, then insert the necessary vertical asymptotes, and finally draw the curve.

Example C: Sketch the graph of $y = 2\tan x$. (Note that the x-axis is scaled by $\pi/2$.)



Since $\cot t = \dfrac{\cos t}{\sin t}$, the graphs of $y = \cos t$, $y = \sin t$ and $y =$ cotangent t are closely related.

Wherever cosine t equals 0, the graph of $y = \cot t$ will also have a y-coordinate equal to 0.

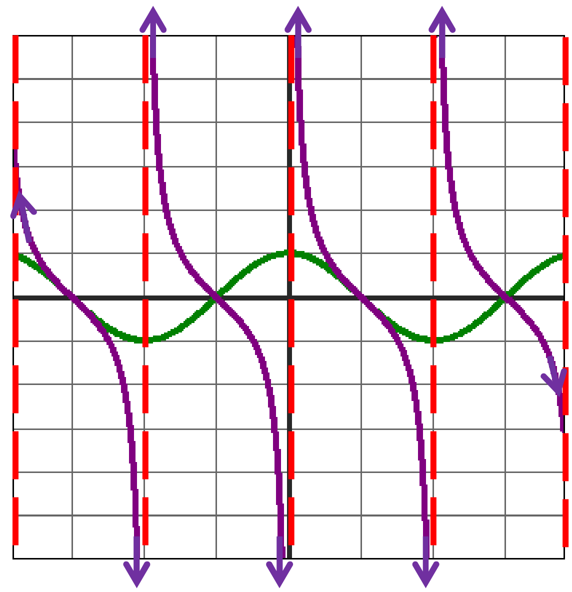
Wherever cosine t equals 1, sine t equals 0, and the graph of $y = \cot t$ has a vertical asymptote.

As sine t approaches 0 [cosine t approaches 1], the graph of $y = \cot t$ will go up or down along a vertical asymptote.

The graph of $y = \cot t$ has neither a maximum nor a minimum.

The period of $y = \cot t$ is π.

Note that x-intercepts are exactly halfway between asymptotes. Also note that asymptotes are exactly halfway between x-intercepts.



My suggestion is that you draw the associated cosine graph first, next put the x-intercepts in place, then insert the necessary vertical asymptotes, and finally draw the curve.

Example D: Sketch the graph of $y = \cot \dfrac{\pi}{2} x$. (Note that the x-axis is scaled by 1.)