# Precalculus 115, section 6.1 Angle Measure & Arc Length

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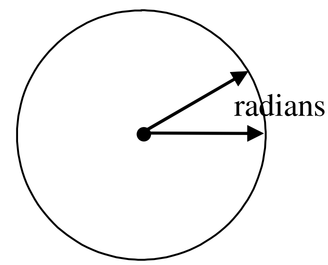
In chapters 5 and 7, we found it much more convenient to measure angles in terms of *radians*. To define radians, we put our angle into a circle so that the vertex is located at the center, and designate the radius of the circle equal to 1. On a Cartesian grid we placed the (vertex of the angle) = (center of the circle) at the origin. This *unit circle* has a circumference (formula $C = 2\pi r$) equal to $2\pi$.

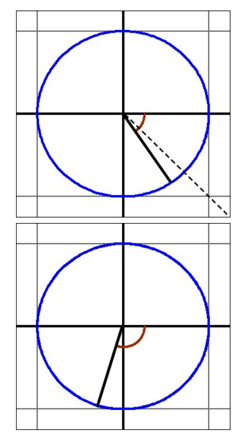
Radian measure was defined as the distance around the circumference of the unit circle one must travel to get from the initial side of the angle to the terminal side. If one travels all the way around the unit circle, it would be an angle of 360º and $2\pi$ radians. This relationship gives us the conversion factor to convert degrees to radians and back again:




$$\text{degrees} * \frac{2\pi}{360} = \text{radians} \quad \text{and} \quad \text{radians} * \frac{360}{2\pi} = \text{degrees}.$$

The text uses the simplest form fractions $\frac{\pi}{180}$ and $\frac{180}{\pi}$.

Example A: Convert –55º to radians and convert $-\frac{7\pi}{12}$ radians to degrees. Identify a positive angle (in radians) with which it is coterminal. *answers:* $-\frac{11\pi}{36}$ *and* $-105°$; $\frac{61\pi}{36}$ *and* $\frac{17\pi}{12}$.

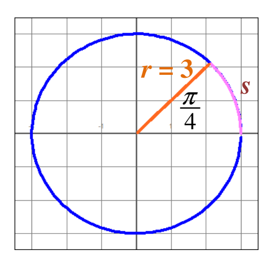


**IMPORTANT**: By convention, when writing trig functions, degree measure is always written with the degree symbol, while radian measure is written without a symbol. So, $\sin(-55°) = \sin\left(-\frac{11\pi}{36}\right)$, while $\sin(-55) = \sin\left(-55 * \frac{180°}{\pi}\right) = \sin\left(-\frac{9900}{\pi}°\right)$.

As part of Lecture 5.2, we encountered the idea of **arc length**. We return to it now in a more formal fashion, and look at the two Examples in more detail.

The circumference formula, $C = 2\pi r$, is linear, and allows us to state that the arc lengths of two different circles are proportional. We can use this fact to find arc lengths in circles with radii not equal to 1.

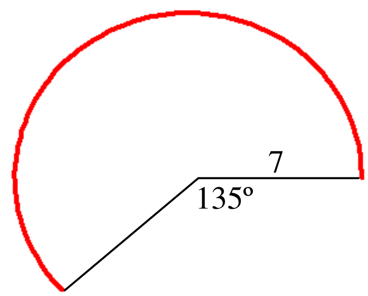
5.2 Example E revisited: In a circle with radius 3, what is the arc length $s$ cut off by a central angle of $\frac{\pi}{4}$ radians? *answer:* $\frac{3\pi}{4}$.



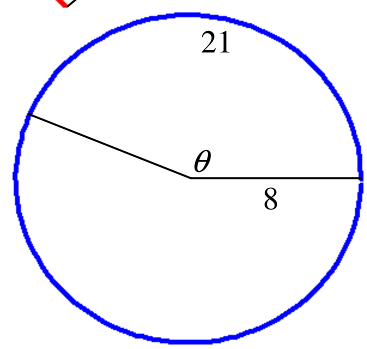

In general, we can state, Arc Length = Radians times Radius. The text states this same idea more formally. "In a circle of radius $r$, the length $s$ of an arc that subtends a central angle of $\theta$ radians is $s = r\theta$."

5.2 Example F revisited: If you are standing on the equator, as the Earth rotates you are actually moving through three-dimensional space, even though it doesn't feel that way. How far do you move in 1 hour? (The radius of Earth at the equator is approximately 4000 miles.) *answer:* $\frac{1000\pi}{3} \approx 1047$ miles.

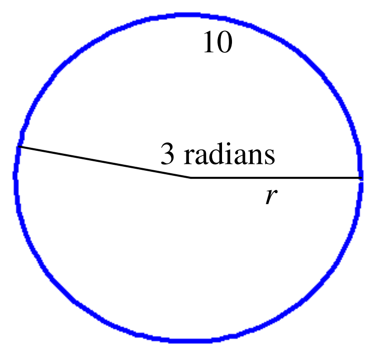
Example B: Find the length $s$ of the arc in the figure to the right.



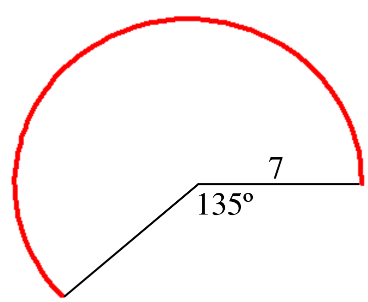

Example C: Find the angle $\theta$ in the figure to the right.




Example D: Find the radius of the circle in the figure to the right.




Example B revisited: Find the area $A$ of the sector in the figure to the right.




"In a circle of radius $r$, the area $A$ of a sector with a central angle of $\theta$ radians is $A = \frac{1}{2}r^2\theta$."

You can either memorize this formula, or you could work out the area of a sector as a fraction of the area of the full circle, as we did for Example B revisited.