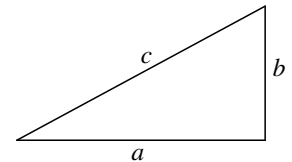


Precalculus 115, section 6.2-6.3 Triangle Ratios

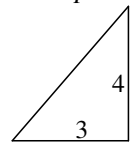
notes prepared by Tim Pilachowski

We began chapter 5 by looking at angles. We began chapter 6 by looking at angles again, but this time focusing more on degree measure rather than radian measure. We now look particularly at the angles found in triangles, specifically right triangles.

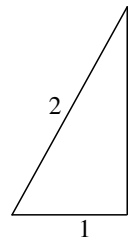
The first thing to remember is the Pythagorean Theorem (which, by the way, goes back much further in time than Pythagoras), which we usually remember as $a^2 + b^2 = c^2$. So, that knowing the lengths of any two sides, we can find the length of the third.



Example A: Given two legs of a right triangle with legs of lengths 3 and 4, find the length of the hypotenuse. *answer: 5* *side note: This combination of numbers, in which all three are integers, is called a Pythagorean triple.*

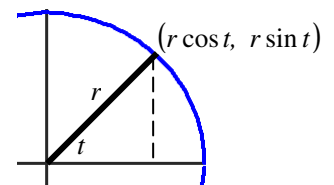


Example B: Given a right triangle with hypotenuse of length 2 and one leg of length 1, find the length of the other leg. *answer: $\sqrt{3}$* *side note: You may recognize this as a $30^\circ, 60^\circ, 90^\circ$ or $\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$ triangle.*



For the moment, we'll go back to circles, with angles in standard position.

In Quadrant I, for each angle t drawn in a circle with radius r , we can draw a right triangle with a side adjacent to the angle with length $r \cos t$, and a side opposite the angle with length $r \sin t$, and a hypotenuse of length r .



This observation leads to the trigonometric triangle ratios.

$$\text{cosine } t = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{r \cos t}{r} = \cos t$$

$$\text{sine } t = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{r \sin t}{r} = \sin t$$

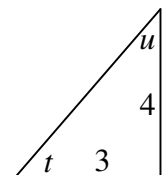
$$\text{tangent } t = \frac{\text{opposite}}{\text{adjacent}} = \frac{r \sin t}{r \cos t} = \tan t$$

A mnemonic sometimes used to remember the trigonometric ratios is SOHCAHTOA.

An advantage of the triangle ratios is that we're no longer limited to considering only angles in standard position on a Cartesian grid. Our right triangle can be drawn in any position that is convenient for us.

Example A revisited: Given the right triangle pictured to the right, find $\sin t$, $\cos t$, $\sin u$, and $\cos u$.

answers: $\frac{4}{5}, \frac{3}{5}, \frac{3}{5}, \frac{4}{5}$



Knowing that the three angles of a triangle add up to 180° , or π radians, we can now state

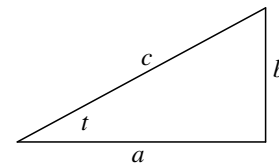
$$\cos\left(\frac{\pi}{2} - t\right) = \sin t \text{ and } \sin\left(\frac{\pi}{2} - t\right) = \cos t.$$

Example C: In the triangle to the right, if $t = 0.7$, and $a = 10.0$, find b and c . Give both the exact answer and an approximation to the nearest tenth.

Make sure your calculator is set for radians before computing the approximation.

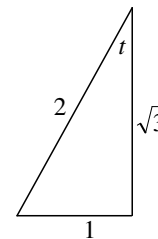
answers: $c = \frac{10}{\cos(0.7)} \approx 13.1$ $b = \sin(0.7) * \frac{10}{\cos(0.7)} \approx 8.4$

Note the difference between the *exact* and the *approximate* answer.



Example B revisited: Given a right triangle with hypotenuse of length 2 and one leg of length 1, find the cosine and sine of the smaller of the two acute angles.

answers: $\cos t = \frac{\sqrt{3}}{2}$, $\sin t = \frac{1}{2}$ side note: The measure of angle t is $30^\circ = \frac{\pi}{6}$ radians.



Example D: Sketch a right triangle that has acute angle θ such that $\csc t = \frac{9}{4}$, then use triangle ratios to find the other five basic trig function values.

In the normal course of events, since we're using right triangles (a geometric figure), we are limited to considering angles whose terminal side lies in Quadrant I when the angle is placed in standard orientation.

In chapters 5 and 7, we used the unit circle definition of trig functions so that we could venture into all four Quadrants, and travel around the unit circle as many or as few times as we wished, and consider both counter clockwise and clockwise rotation. By always being aware of which Quadrant we were in, we could keep track of which trig function values were positive and which were negative.

It's sometimes a bit risky, but if you can keep track of Quadrants, along with positives and negatives, you can use triangle ratios to determine the other five trig function values given any one trig function value.

5.2 Example C revisited: Given $\tan t = -\frac{3}{2}$ and t in Quadrant II, use triangle ratios to find the values of the other five trigonometric functions.