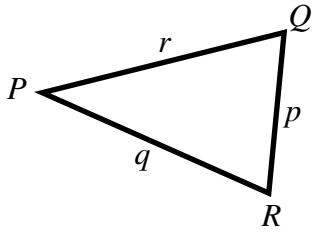
Precalculus 115, section 6.6 The Law of Cosines

notes prepared by Tim Pilachowski

The Law of Sines works for us, as long as we know any angle and the length of the side opposite it in our triangle. In a case where we know two sides and an included angle, or know the three sides but no angles, we need something different.

The Law of Cosines. Given a triangle with angles P, Q and R, with sides opposite those angles p, q and r respectively, $p^2 = q^2 + r^2 - 2pq\cos R$.

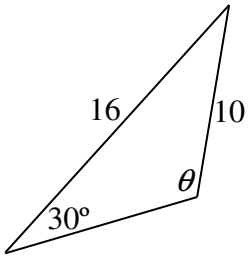


The text has the proof, so we won't duplicate it here. Check it out for yourself. The text has three versions, using the letters a, b, and c, but all you really need to know is the structure. We'll simply name angles and side-lengths in whatever fashion is most convenient.

Example A: Given $\angle A = 45°,\ b = 15,\ c = 24$, find the values of a, cos B, and cos C.

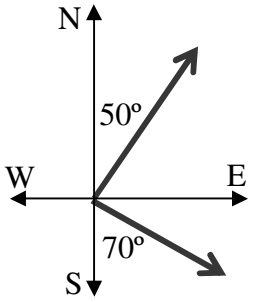
Example B: Given $a = 20,\ b = 10,\ c = 22$, find the measure of $\angle A$.

Example C: Find the measure of θ in the picture to the right below.



[picture is not drawn to scale]

Example D (text # 44): Two boats leave the same port at the same time. One travels at a speed of 30 mi/h in the direction N 50º E and the other travels at a speed of 26 mi/h in a direction S 70º E. How far apart are the boats after 30 minutes?



[picture is not drawn to scale]