Precalculus 115, section 7. 1 Trigonometric Identities

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For this section we'll be relying on the definitions of the four "other than sine and cosine" functions, along with the Ptyhagorean identity:

$$\sec t = \frac{1}{\cos t} \qquad \csc t = \frac{1}{\sin t} \qquad \tan t = \frac{\sin t}{\cos t} \qquad \cot t = \frac{\cos t}{\sin t} \qquad \sin^2 t + \cos^2 t = 1.$$

Our goal will be to simplify expressions and to prove identities.

Recall from Lecture 1.4 on simplifying rational expressions that we had to state restrictions on the variable. We'll do the same thing when working with trigonometric identities.

One last thing: Our method for this section will be to use the definitions and identities above to rewrite everything in terms of sine and cosine, then cancel and simplify wherever possible.

Example A: Simplify the trigonometric expression $\dfrac{\tan x}{\sec(-x)}$. State all restrictions on the variable.

Example B: Simplify the trigonometric expression $\dfrac{\csc x - \sin x}{\cot x}$. State all restrictions on the variable.

Example C: Verify the identity $\dfrac{\tan x \csc x}{\sec x} = 1$. State all restrictions on the variable.

Example D: Verify the identity $\cos(-x) - \sin(-x) = \cos x + \sin x$. State all restrictions on the variable.

Example E: Verify the identity $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$. State all restrictions on the variable.

Example F: Verify the identity $\dfrac{\csc x + \sec x}{\cot x + \tan x} = \sin x + \cos x$. State all restrictions on the variable.

Example G: Verify the identity $\dfrac{1 + \csc^2 x}{1 + \cot^2 x} = 1 + \sin^2 x$. State all restrictions on the variable.