Calculus 120, section 1.1 The Slope of a Line

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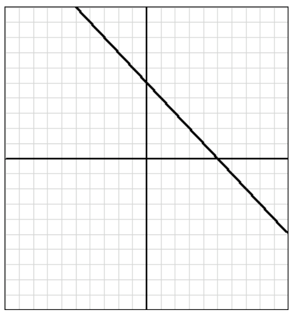
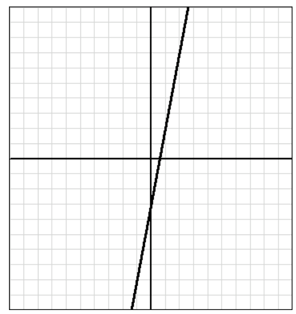
Bring your graphing calculator to the next Lecture.

You might ask, "Why are we talking about lines again? I did this in high school, and again in Math 113. Haven't we said everything there is to say?" The answer lies in knowing that it is not the lines themselves which are of interest, but rather it is their slopes. Slope is a measurement of change, and Calculus is the mathematics of change. If we can look at the rate of change in a situation, determine whether the rate of change is constant or whether it fluctuates, then we can use those observations to find a mathematical model (i.e. equation) to describe the situation. The model can then be used not only to evaluate the past but also to predict and plan for the future.

Examples A & B: Consider the following two graphs of linear equations $x+y=5$ and $y=5x-3$:





By defining slope as $\frac{\text{change in } y}{\text{change in } x}=\frac{\Delta y}{\Delta x}$ we address both direction and steepness. *Answers*: -1; 5

Example C: slope of a horizontal line and slope of a vertical line *Answers*: 0; undefined

You'll also need to know about equations of horizontal and vertical lines.

Examples A & B extended: slope-intercept form of the equation: $f(x)=mx+b$

Example D: $3x+2y=7$ *Answer*: $-\frac{3}{2}$

Example E: If a new car costs $24000, and its value depreciates (goes down) by $2000 each year. Find a mathematical model (i.e. equation). *Answer*: $-2x + 24$

Example E extended: If you only have $10000 to spend on a used car, about how old will it be to still be in your price range? *Answer*: 7 years

Example F: Find the equation of the line which passes through the points $(-5, 3)$ and $(-10, 0)$.
Answer: $y=\frac{3}{5}x+6$

Example G: When a company spent $10000 on radio ads, they had revenue of $8,500,000. When they spent $20000, their revenue rose to $9 million. Find the mathematical model for revenue as a function of amount spent on advertising. *Answer*: $0.05x + 8$

Example H: Find an equation to relate C° to F°. *Answer*: $F^\circ = \frac{9}{5}C^\circ + 32$ or $C^\circ = \frac{5}{9}\left(F^\circ - 32\right)$

Example I: Parallel lines will necessarily have equal slopes—the "over and up" or "over and down" movements must match since the two lines go in the same direction. You learned that perpendicular lines have slopes that are negative reciprocals. Must this be true? Consider the line $y = 2x$ and its perpendicular. With a little basic geometry and the knowledge that the three angles of a triangle must add up to 180º, it's not too hard to show that the "over 1 up 2" triangle of $y = 2x$ is transposed into an "over 2 down 1" triangle for the perpendicular line.. So, while the slope of $y = 2x$ is $m = \frac{2}{1} = 2$, the slope of the perpendicular must be $m = \frac{-1}{2} = -\frac{1}{2}$, the negative reciprocal. The same principles can be used to prove the relationship is true for any perpendicular lines.