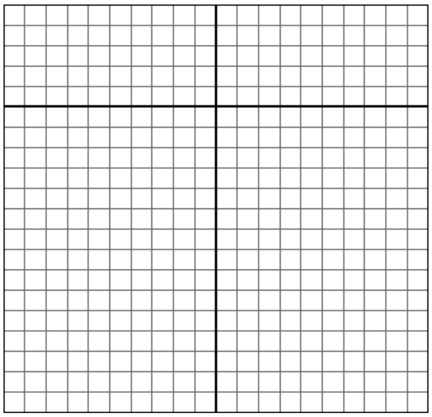
Calculus 120, section 2.3 Sketching a Curve, Part 1

notes by Tim Pilachowski

Section 2.2 examined first and second derivative attributes of a graph. Now we get to begin with the equation and work toward its graph. In this section we'll only work with f', f'', extrema, and concavity.

Example A: Sketch the graph of $f(x) = x^2 + 5x - 6$.

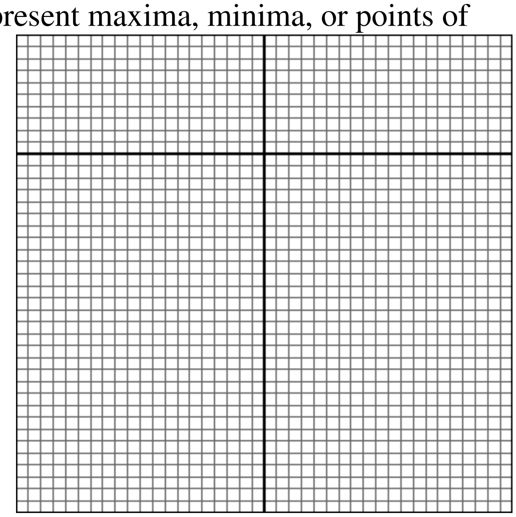
Using a table of signs we can determine the slope of the curve on the intervals on either side of the critical value, and thus determine whether this value represents a maximum, minimum, or point of inflection.



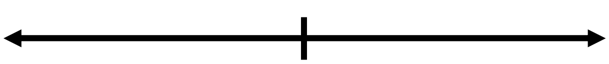
interval			
sign of f'			

Example B: Sketch the graph of $f(x) = x^3 - 3x^2 - 9x + 1$.

Using the factors of f' we can determine the slope of the curve on the intervals on either side of and in between these two critical values to determine whether these values represent maxima, minima, or points of inflection.



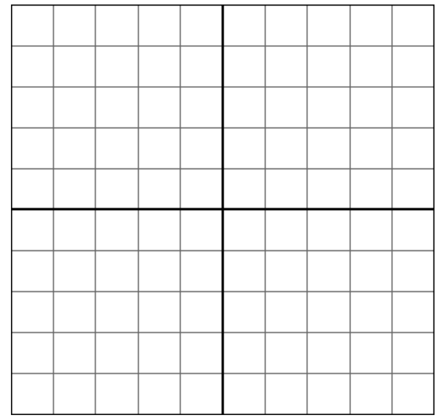
The factors of f'' enable us to determine concavity.



Example C: Sketch the graph of $f(x) = (1-2x)^3$.

Using the factors of f' we can determine the slope of the curve on the intervals on either side of the critical value, and thus determine whether this value represents a maximum, minimum, or point of inflection.

		*		=	
		*		=	
		*		=	
		*		=	

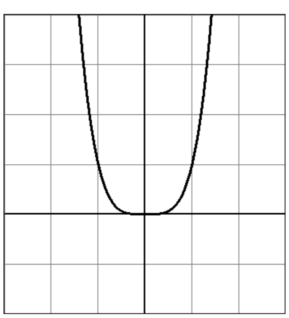


The factors of f'' enable us to determine concavity.

		*		=	
		*		=	
		*		=	
		*		=	

Example D. Find an example for which $f'' = 0$ does *not* imply a point of inflection.

A table of signs indicates the nature of the curve at the origin:



interval			
sign of y			
sign of $\frac{dy}{dx}$			
sign of $\frac{d^2y}{dx^2}$			
behavior of graph			

$f' > 0$ implies increasing function; increasing function does not imply $f' > 0$ at all points, e.g. $y = x^3$.
$f'' > 0$ implies a graph is concave up; a graph concave up does not imply $f'' > 0$ at all points, e.g. $y = x^4$.
Similar statements can be made with regard to decreasing functions and graphs that are concave down.