Calculus 120, section 2.5 Optimization Problems (Applications)

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Oh boy—now we get to start on word problems! First, a definition: optimize – (verb) to make as perfect or effective as possible. In calculus terms, for anything optimal, we will be searching for some sort of *maximum* or *minimum*.

Example A: The function $s(t) = -16t^2 + 10t + 240$ calculates the height of an object, s, after time, t, thrown upward at 10 feet per second from a bridge which is 240 feet above the river below. In section 1.8 lecture notes we asked:

a) What is the height of the rock after 2 seconds? answer: Find $s(2) = 196$ feet.

b) How much height did the rock gain after 2 seconds? answer: Calculate $s(2) - s(0) =$ loss of 44 feet of height.

c) What is the velocity of the rock after 2 seconds? answer: Find $s'(2) =$ the rock is falling at 54 feet per second.

d) What is the average velocity during the first 2 seconds? answer Find $\frac{s(2) - s(0)}{2 - 0} =$ on average the rock has fallen at 22 feet per second.

New questions:

e) How long does it take for the rock to reach its maximum height? *Answer*: $\frac{5}{16}$ sec.

f) What is the maximum height above the water reached by the rock? *Answer*: $241 \frac{9}{16}$ feet

g) What is the maximum height above the bridge reached by the rock? *Answer*: $1 \frac{9}{16}$ feet

Example B: Public health officials use rates of change to quantify the spread of an epidemic into an equation, which they then use to determine the most effective measures to counter it. A recent measles epidemic followed the equation $y = 45t^2 - t^3$ where y = the number of people infected and t = time in days.

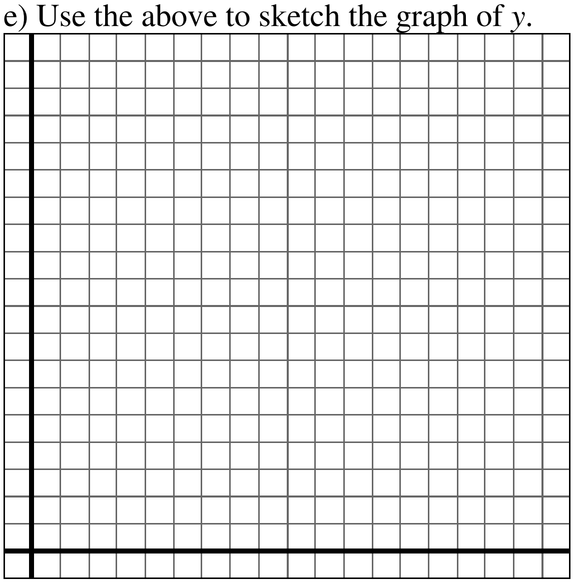
a) What is the domain of this function? *Answer*: $0 \le t \le 45$ days

b) How many people are infected after 5 days? *Answer*: 1000 people

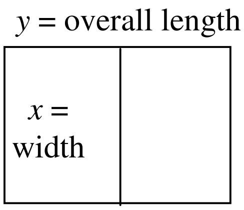
c) What is the rate of spread after 5 days? *Answer*: 375 new cases per day

d) After how many days does the number of cases reach its maximum? *Answer*: 30 days

e) Use the above to sketch the graph of y.



Example C: Perhaps you have already encountered versions of the infamous corral problem favored by Math 113 and 120 course coordinators. A farmer has 900 feet of fencing with which to build a pen for his animals, and being a frugal sort doesn't want to buy any more fencing. He needs two pens, but can build them adjacent to each other, sharing one side as in the diagram to the left. Find the dimensions that will give the maximum area. *Answer*: 225 ft long by 150 ft wide



Example D: Optimization does not always involve a maximum. The fuel, maintenance and labor costs (in dollars per mile) of operating a truck on an interstate highway are described as a function of the truck's velocity (miles per hour) by the algebraic rule $C(v) = 78 + 1.2v + 5880v^{-1}$. What speed should the driver maintain on a 600 mile haul to minimize costs? *Answer*: 70 mph.