Calculus 120, section 2.6 Further Optimization Problems (More Applications)

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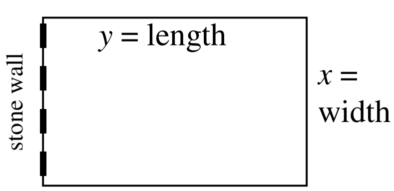
To pick up where we left off last time: finding maxima or minima is often an objective in the real world. Today we take a look at some business-related applications.

Example A: *In this example, it will be important to know which amount is being optimized*. An efficiency study of the 8 am to noon shift at a factory shows that the number of units, N, produced by an average worker t hours after 8 am is modeled by the function $N(t) = -t^3 + 9t^2 + 12t$. Note that the domain is $0 \le t \le 4$.

Question 1: At what time is the *number of units produced* at its peak? *Answer*: noon

Question 2: At what time is the *rate of production* at its peak? *Answer*: 11 am

One prime goal in business is to keep costs to a minimum—rent or mortgage, insurance, salaries, utility bills, expense for purchasing material, advertising, etc. In each example below there will be a function which we want to minimize or maximize: the *objective function*. The limitations of reality become *constraint functions*.



Example B: Farmer Al needs to fence in 800 yd^2, with one wall being made of stone which costs \$24 per yard, and the other three sides being wire mesh which costs \$8 per yard. What dimensions will minimize cost?
Answer: 40 yd long by 20 yd wide

Our next example involves minimizing inventory costs: ordering costs (the money paid for shipping purchased items) and carrying costs (the money spent on insurance, maintenance and taxes to keep items in storage and available for when we might need them).

Example C: For each delivery, a company pays a $50 shipping cost. Carrying costs (based on the average number of units in stock) are $4 per unit per year. If we assume sales are uniform over the course of a year, and we expect sales of 2500 units, how often should we place an order so as to minimize costs?

Scenario 1) If we order all 2500 units at one time the ordering cost will be $50. If we sell them at a steady rate during the year, the average number in stock will be $2500 \div 2 = 1250$, so the annual carrying cost will be $1250(4) = \$5000$, giving a total cost of $5050.

Scenario 2) If we make four equally-sized orders of 625, the ordering costs will be $4(50) = \$200$. Average inventory will be $625 \div 2 = 312.5$, so annual carrying cost will be $312.5(4) = \$1250$. Total cost is $1450.

Scenario 3) We'll derive an equation and use calculus to find the means of ordering which will minimize cost. First we define variables. Let C = total cost.
Let r = the number of orders placed during the year; ordering costs will be $50r$.

Let x = the number of units ordered at a time; average number in stock will be $\frac{x}{2}$; carrying costs are $\frac{x}{2}(4) = 2x$.

We want to minimize our *objective function*.

We are also working under a *constraint*: we want total number of items ordered to equal out projected sales.
Final answer: Order 250 units, 10 times per year.

Example D: A company manufactures 6000 units per year. We will assume sales are uniformly distributed throughout the year. Set-up costs are $60 for each production run. Carrying costs (based on the average number of units in stock) are $8 per unit annually. Find the number of units to manufacture in each production run which will minimize costs.
Answer: 300 units each in 20 production runs
Note: This example is actually the same as the previous one. The only difference is that it is seen from the point-of-view of a manufacturer rather than a distributor.