Calculus 120, section 2.7 Business & Economics (Still More Applications)

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In this section we'll focus on an extended example that connects price, revenue, costs and profits.

Example A: Based on past sales numbers, when the price of a doohickey was set at $130 the company sold 1000 each month. When they lowered their price to $100, the demand went up to 4000. Use this information to derive an equation for price as a function of demand. *Answer*: $p(x) = -0.01x + 140$

Example A extended: Find and interpret the *y*- and *x*-intercepts. *Answer*: (0, 140), (14000, 0)

Example B: Using the price equation from Example A, derive an equation for revenue.
Answer: $R(x) = -0.01x^2 + 140x$

Example B extended: Derive an equation for marginal revenue then determine the maximum possible revenue.
Answer: $490000

Keep in mind that revenue is not profit. Costs must be subtracted from revenue to calculate profit, and the maximum revenue will not necessarily yield the maximum profit.

Example C: The same company incurs costs when manufacturing doohickeys. Fixed costs (rent, management salaries, utilities, security, insurance) are $15,000 per month. Labor and materials cost $40 for each doohickey. Use this information to derive an equation for cost as a function of number produced.
Answer: $C(x) = 40x + 15000$

As a side note, in the real world, a cost function is more usually derived from more complicated factors, and will often be at least a quadratic function.
Example C extended: Derive and interpret an equation for marginal cost. *Answer*: $C'(x) = 40$

Example D: Derive an equation for profit made from making and selling doohickeys.
Answer: $P(x) = -0.01x^2 + 100x - 15000$

Example D extended: Find the level of production which will yield maximum profit. *Answer*: 5000

Note that we could also have solved this problem by solving the equation $R'(x) = C'(x)$.

Example E: At what price should our company sell doohickeys to maximize profit, and what are the associated Cost, Revenue and Profit amounts? *Answer*: $90; $215,000; $450,000; $235,000.