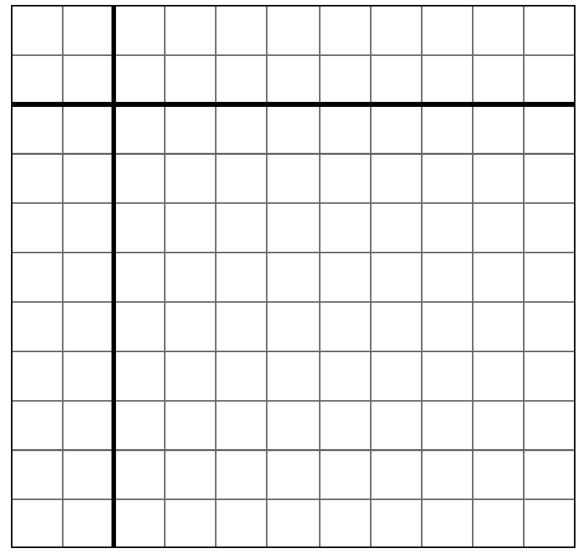
Calculus 120, section 4.3 Differentiation of e^x

notes by Tim Pilachowski

Recall from last time that the natural logarithm function, $y = e^x$, is its own derivative. That is, $\frac{d}{dx}\left(e^x\right) = e^x$.

Example A: Sketch the graph of $f(x) = \frac{x^3}{e^x}$.



domain:

y-intercept:

x-intercept:

first derivative:

possible extrema and points of inflection:

vertical asymptotes:

second derivative:

other possible points of inflection:

concavity of *f*:

interval	$x < 0$	$0 < x < 3-\sqrt{3}$	$3-\sqrt{3} < x < 3+\sqrt{3}$	$x > 3+\sqrt{3}$
sign of f''				
concavity of *f*				

horizontal asymptotes:

In Example A we used the quotient rule. Sometimes the chain rule is needed for exponential functions.

Example B: Given $h(x) = e^{x^2-x}$, find the first derivative. *Answer*: $\left(e^{x^2-x}\right)(2x-1)$

Example B extended: Given $h(x) = e^{x^2-x}$, find the second derivative. *Answer*: $2\left(e^{x^2-x}\right)+\left(e^{x^2-x}\right)(2x-1)^2$

Example B extended once more: Given $h(x) = e^{x^2-x}$, find the location of any extrema. *Answer*: $\left(\frac{1}{2}, \frac{1}{\sqrt[4]{e}}\right)$

In general, we can apply the chain rule to functions of the form $y = Ce^{kx}$:

$$y' = Ce^{kx} * \frac{d}{dx}(kx) = Ce^{kx} * k = kCe^{kx}.$$

Example C: Let $g(t) = e^{2t} + e^{-2t}$. Find the possible locations of relative extrema. *Answer*: $t = 0$

Example D: The accumulated amount of an investment of $100 with 5% annual interest compounded continuously for t years is given by the formula $A = 100e^{0.05t}$. Find the first derivative. *Answer*: $5e^{0.05t}$

From the observation and example above, we can conclude that a differential equation of the form $y' = ky$ must have an exponential base e as its solution, i.e. $y = Ce^{kx}$ for some C. No other function is its own derivative. As a side note, in solving a differential equation like this we are essentially finding an anti-derivative, the "reverse" of finding a derivative.

Example E: Determine all functions $y = f(x)$ such that $y' = \frac{1}{3}y$. *Answer*: $y = Ce^{\frac{1}{3}x}$

Example E extended: Determine the function $y = f(x)$ such that $y' = \frac{1}{3}y$ and $y(0) = 7$. *Answer*: $y = 7e^{\frac{1}{3}x}$

One last note: An exponential function in a base other than e, for example $y = 2^x$, can be easily rewritten as an exponential function base e. We'll have the tools with which we'll do the conversion in the next section.