Calculus 120, section 6.1 Antidifferentiation

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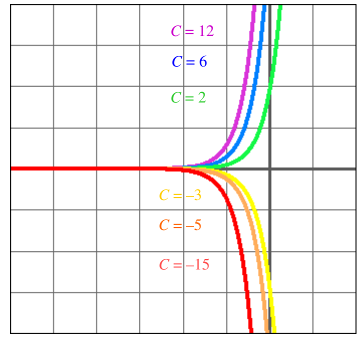
In mathematics we often go backwards. Subtraction is the "backwards" of addition: $2 + 3 = 5$ and $5 - 3 = 2$. Division is the "backwards" of multiplication: $4 * 5 = 20$ and $20 \div 5 = 4$. Solving is the "backwards" of calculation: $3(2 + 5) = 20$ and $3(x + 5) = 20$.

An integral, or antiderivative, is the "backwards" of finding the first derivative. In the real world, we may know something about the rate of change from experiment or observation, and want to work our way backwards to find the equation that describes a phenomenon.

Example A: Given $y' = 3y$, find y.



The derivative of our solution satisfies the original differential equation, although without further information, we cannot specify the value of the coefficient C.

Example B: Given a function $f(x) = 5x^4$ find a function $F(x)$ such that $F'(x) = f(x)$.

Example B extended: Given a function $f(x) = 5x^4$ find a function $F(x)$ such that $F'(x) = f(x)$ and $F(1) = 8$.

In Example B above, the question was phrased, "Find a function $F(x)$ such that $F'(x) = f(x)$." There are four other equivalent ways to ask the same thing:

Find the antiderivative of $f(x)$. Integrate $f(x)$.

Find the integral of $f(x)$. Find $\int f(x)\,dx$.

Example C: Find the antiderivative of $f(x) = x^4$.

Example C another way: Find the antiderivative of $f(x) = x^4$.

The power rule for derivatives says “multiply times the exponent and subtract 1 to get the new exponent”.
An anti-derivative is the opposite of a derivative.
We will “add 1 to the exponent and divide by the new exponent”.

From this example, we can generalize the process for integrating power functions:

$$\int x^r \, dx = \frac{1}{r+1} x^{r+1} + C, \quad r \neq -1.$$

Note the restriction on r. We have to avoid a 0 in the denominator since division by 0 is undefined.

Example D: Evaluate $\int \frac{1}{\sqrt{1-3x}} \, dx$.

The general power rule for derivatives says “multiply times the exponent, subtract 1 to get the new exponent, then multiply by the derivative of the inside”.
An anti-derivative is the opposite of a derivative.
We will “add 1 to the exponent, divide by the new exponent, then divide by the derivative of the inside”.

Example E: Integrate $f(x) = x^{-1}$.

The "power rule for integrals" can't be used here, because of the restriction on r. For this one, you'll have to remember back to section 4.5.

The absolute value brackets are needed to retain the domain of x^{-1}: $-\infty < x < 0$ and $0 < x < \infty$. Also, don't forget that the " $+ C$ " constant of integration is necessary.
In WebAssign, a small c instead of a capital C will be marked as a wrong answer.

Now is as good a time as any to point out the "dx" part of the integral $\int f(x)\,dx$. It is a *necessary* part of any integral, since we are finding the antiderivative "with respect to x":

$$f(x) = \frac{d}{dx}[F(x)] \quad \Leftrightarrow \quad \int f(x)\,dx = F(x) + C.$$

Example F: Find the integral of $f(x) = e^{-2x}$.

Example F another way: Find the integral of $f(x) = e^{-2x}$.

The derivative rule for exponential functions $y = e^{kx}$ says "multiply times k".
An anti-derivative is the opposite of a derivative.
We will "divide by k".

From this example, we can generalize the process for integrating exponential growth/decay functions:

$$\int e^{kx} = \frac{1}{k}e^{kx} + C, \quad k \neq 0.$$

Note the restriction on k. We have to avoid a 0 in the denominator since division by 0 is undefined.

Example G: Find $\int e^0 \, dx$.

Note that in this example, as in all the others, we can easily check our answer by finding its derivative.

Example H: Evaluate $\int \left(3x^{-6} + 2e^{5x} + 4(1-2x)^{-1} + 7\right) dx$.

As a side note, while the parentheses surrounding the function may be helpful they are technically not necessary. The integral "begins" at the $\int$ symbol and "ends" at the *dx*. Everything in-between is included.

Example I: A company determines that its marginal revenue, in dollars, is expressed by the equation $R'(x) = -0.03x^2 + 500x - 12000$ where x is the number of units sold. They also know that if they have no sales they still must pay operating costs of \$23000. Find their revenue equation.