May 9, 2005 Math 220 mini-Test [7.3 – 7.5] Pilachowski **ANSWER KEY**

Put your name and your TA's name on the top of the answer page.
Don't take time to rewrite the question. Go right to your answer.

1. [10] Find all possible relative maximum and minimum points of $f(x, y) = -x^2 - y^2 + 6x + 8y - 21$ and then determine whether the points you found are maximum, minimum, or neither.

$\frac{\partial f}{\partial x} = -2x + 6$, which equals 0 where $x = 3$. $\frac{\partial f}{\partial y} = -2y + 8$, which equals 0 where $x = 4$. So there is only one possible relative maximum or minimum. Apply the second derivative test to determine which.

$$\frac{\partial^2 f}{\partial x^2} = \frac{\partial}{\partial x}(-2x + 6) = -2, \quad \frac{\partial^2 f}{\partial y^2} = \frac{\partial}{\partial y}(-2y + 8) = -2, \quad \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y}(-2x + 6) = 0$$

So $D(x, y) = (-2)(-2) - 0^2 = 4 > 0$ for all values of x and y, including $x = 3$ and $y = 4$.

Since $\frac{\partial^2 f}{\partial x^2} = -2 < 0$ for all values of x and y, including $x = 3$ and $y = 4$, we conclude $f(3, 4)$ is a relative maximum.

2. [10] Use Lagrange multipliers to find the minimum value of $f(x, y) = x^2 + y^2$ subject to $2x + y = 10$.

First set constraint $g(x, y) = 2x + y - 10 = 0$.
Then $F(x, y, \lambda) = x^2 + y^2 + \lambda 2x + \lambda y - \lambda 10$.

$$(1)\frac{\partial F}{\partial x} = 2x + 2\lambda = 0, \quad (2)\frac{\partial F}{\partial y} = 2y + \lambda = 0, \quad (3)\frac{\partial F}{\partial \lambda} = 2x + y - 10 = 0.$$

Solve (1) and (2) as a system: $\begin{cases} 2x + 2\lambda = 0 \\ 2y + \lambda = 0 \end{cases} \to \begin{cases} \lambda = -x \\ \lambda = -2y \end{cases} \to -x = -2y \to x = 2y$

Substitute into (3): $2(2y) + y - 10 = 0 \to 5y = 10 \to y = 2$, and thus $x = 4$.
The minimum value of f is $f(4, 2) = 4^2 + 2^2 = 20$.

3. [10] Find the best least-squares fit (i.e. regression line equation) for the points (1, 4), (2, 5) and (3, 8).

Two methods are outlined below. The first finds the minimum of the error^2 function.

x	observed y	regression y	error^2	
1	4	$A + B$	$(4 - A - B)^2$	
2	5	$2A + B$	$(5 - 2A - B)^2$	
3	8	$3A + B$	$(8 - 3A - B)^2$	minimize $F = (4 - A - B)^2 + (5 - 2A - B)^2 + (8 - 3A - B)^2$

$$\frac{\partial F}{\partial A} = 2(4 - A - B)(-1) + 2(5 - 2A - B)(-2) + 2(8 - 3A - B)(-3) = -76 + 28A + 12B = 0$$

$$\frac{\partial F}{\partial B} = 2(4 - A - B)(-1) + 2(5 - 2A - B)(-1) + 2(8 - 3A - B)(-1) = -34 + 12A + 6B = 0$$

$$\begin{cases} 28A + 12B = 76 \\ 12A + 6B = 34 \end{cases} \to \begin{cases} 14A + 6B = 38 \\ -12A - 6B = -34 \end{cases} \to 2A = 4 \to A = 2 \to 12(2) + 6B = 34 \to 6B = 10 \to B = \frac{5}{3}.$$

The best-fit equation is $y = 2x + \frac{5}{3}$.

alternate method: Use the formulas developed in lecture and in the text. Note that $N = 3$.

	x	y	xy	x^2
	1	4	4	1
	2	5	10	4
	3	8	24	9
$\Sigma =$	6	17	38	14

$A = \frac{3*38-6*17}{3*14-(6)^2} = \frac{114-102}{42-36} = \frac{12}{6} = 2$ $\qquad B = \frac{17-2*6}{3} = \frac{17-12}{3} = \frac{5}{3}$.

The best-fit equation is $y = 2x + \frac{5}{3}$.