# Calculus 140, section 3.7 Related Rates

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In section 3.6 we considered equations (which may or may not have been functions) relating two variables x and y and used implicit differentiation to find $\dfrac{dy}{dx}$.

This section takes the process of implicit differentiation and applies it in a new context. We'll once again have equations relating x and y, but instead of finding the rate at which x and y are changing with respect to each other, we'll be finding the rate of change of each with respect to a third variable, t (almost always representing time.

Example A-1: A go-kart track is in the shape of an ellipse with equation $x^2 + 4y^2 = 2500$. Go-karts travel counterclockwise around the track, starting at point (50 ,0). At the end of the first lap [i.e. at the point (50, 0) again] Rodna's go-kart is going 10 mph [i.e. $\dfrac{dy}{dt} = 10$]. How is her x-coordinate changing [i.e. what is the value of $\dfrac{dx}{dt}$]?   *answer:* ≈ 0 mph
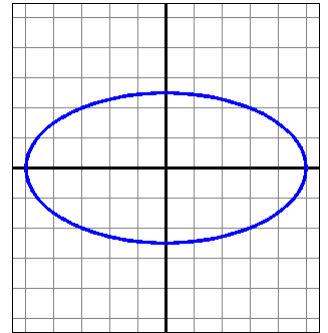
Example A-2: A go-kart track is in the shape of an ellipse with equation $x^2 + 4y^2 = 2500$. Lewce passes Rodna at the point (30 ,20). At that moment, Lewce's y-velocity is 6 mph [i.e. $\dfrac{dy}{dt} = 6$]. How is his x-coordinate changing [i.e. what is the value of $\dfrac{dx}{dt}$]?   *answer:* ≈ -16 mph
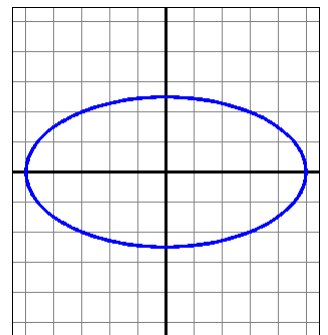
Example B (a classic application): A 24-foot extension ladder is leaning against the side of the house, and I'm standing on a rung 1 ft from the top of the ladder so I can wash windows on the second floor. My brother was supposed to be holding the bottom, but got distracted by the ice-cream truck. When he walked away, the bottom of the ladder was 10 ft away from the house. At the time the bottom of the ladder reaches 15 ft away from the house it was slipping at a rate of 2 ft per second. How fast am I (near the top of the ladder) dropping toward the ground?   *answer*: ≈ -1.72 ft/s
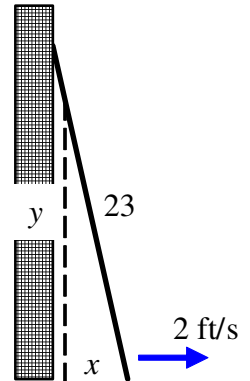
Example C: An oil tanker has run aground and is leaking crude oil. Assuming (simplistically) calm waters, the oil slick will be circular. a) If the radius of the oil slick is increasing at a rate of 0.4 feet per minute when the radius is 400 feet, how is the area of the slick changing with respect to time? b) If the depth of the oil on the water is 3 in, what is the rate at which the oil is spilling (i.e. how is the volume changing with respect to time.)? c) Given 1 ft$^3 \approx 7.48$ gal, Determine how much oil has spilled at this time, and the rate at which the oil is leaking in gallons per minute.   *answers:* ≈ 1005 ft^2; ≈ 251 ft^3; $\approx 914{,}831$ gal, ≈ 1879.9 gal/min

Example D: An animal's *basal metabolic rate* is the rate of heat by an animal in a specified unit of time. Empirical data indicate that the basal metabolic rate of a warm-blooded animal with mass k kg can be expressed by $m = 70k^{0.75}$ kilocalories per day. If an 8 kg marmot is losing 1% of its mass per day, what is the rate of change of the basal metabolic rate?   *answer:* ≈ -2.50 kilocalories per day per day

Example E. Environmental scientists have determined that the relationship between levels of a particular pollutant L and population size p (defined in thousands) can be expressed by $L(p) = p^2 + e^{0.01p} + 1000$. Assuming a population of 400,000 which is increasing at a rate of 2000 per year, how is the level of the pollutant changing?   *answer:* $\frac{dL}{dt} \approx 1601$ units per year