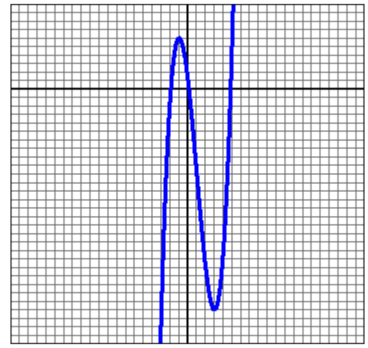
Calculus 140, section 4.5 First and Second Derivative Tests

notes by Tim Pilachowski

Reminder: You will not be able to use a graphing calculator on tests!

Example A: Find all relative extreme values of $f(x) = x^3 - 3x^2 - 9x + 1$.



first derivative:

critical numbers:

Using the factors of f' we can investigate the intervals on either side of and in between these two critical values interval(s) to determine where $f'(x) > 0$ and where $f'(x) < 0$.

relative maximum value(s):

relative minimum value(s):

The text defines "relative maximum value", "relative minimum value" and "relative extreme value" on an open interval in Definition 4.9.

Theorem 4.10 [The First Derivative Test]: "Let *f* be differentiable on an open interval about the number *c* except possibly at *c*, where *f* is continuous.

a. If f' changes from positive to negative at *c*, then *f* has a relative maximum value at *c*.

b. If f' changes from negative to positive at *c*, then *f* has a relative minimum value at *c*."

The book's proof takes five lines of text.

Example B (see section 4.1 Example C): Consider the function $f(x) = \dfrac{x^3}{e^x}$.

first derivative:

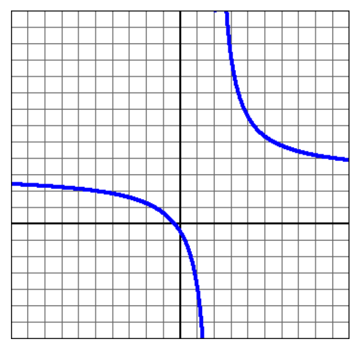
critical numbers:

Example C: Given $f(x) = \cos x + \frac{\sqrt{2}}{2} x$, determine values c where $f'(x)$ changes from negative to positive, or from positive to negative.

first derivative:

critical numbers:

Example D: Consider the function $f(x) = \frac{3x+1}{x-2}$.



first derivative:

critical numbers:

Theorem 4.11 [The Second Derivative Test]: "Assume that $f'(c) = 0$ and that $f''(c)$ exists.

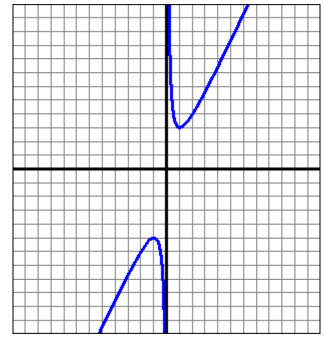
a. If $f''(c) < 0$, then $f(c)$ is a relative maximum value of f.

b. If $f''(c) > 0$, then $f(c)$ is a relative minimum value of f.

If $f''(c) = 0$, then from this test alone we cannot draw any conclusions about a relative extreme value of f at c."

Take a look at the text's proof, especially the examples for which both $f'(c) = 0$ and $f''(c) = 0$, but f has neither a maximum nor a minimum.

Example E: Consider the function $f(x) = 2x + \frac{2}{x} - 1 = 2x + 2x^{-1} - 1$. Use the Second Derivative Test to determine any relative extreme values.



first derivative:

critical numbers:

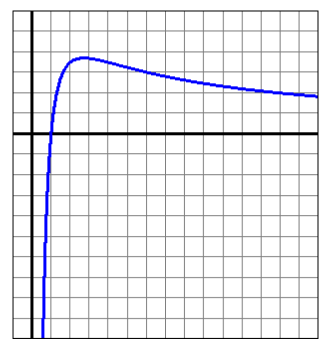
second derivative:

second derivative test:

relative maximum value(s):

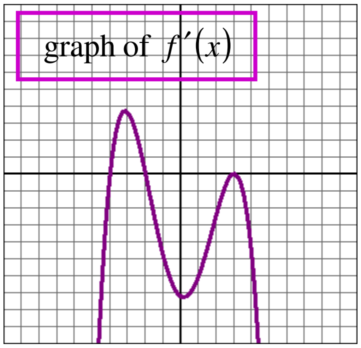
relative minimum value(s):

Example F: Given $f(x) = \frac{10 \ln x}{x}$, determine any relative extreme values.



first derivative:

critical numbers:



Example G: Without knowing the function itself, describe the behavior of its graph only using information provided by its first derivative. The graph to the left is a graph of $f'(x)$.

critical numbers:

Note that, since we don't have a formula for *f*, we cannot determine y-coordinates of the critical points.

Interval	$x < -4$	$x = -4$	$-4 < x < -2$
value of f'			

Since the first derivative (slope of *f*)…

Interval	$-4 < x < -2$	$x = -2$	$-2 < x < 3$
value of f'			

Since the first derivative (slope of *f*) …

interval	$-2 < x < 3$	$x = 3$	$3 < x$
value of f'			

Since the first derivative (slope of *f*) …

interval(s) increasing:

interval(s) decreasing:

Putting all of the information above together, we can draw a preliminary sketch of the graph for *f*.