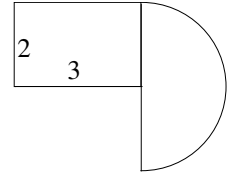


Calculus 140, section 5.8 Another Look at Area

notes by Tim Pilachowski

Example A: Find the area of the geometric figure pictured to the right. *answer: $6 + 2\pi$*

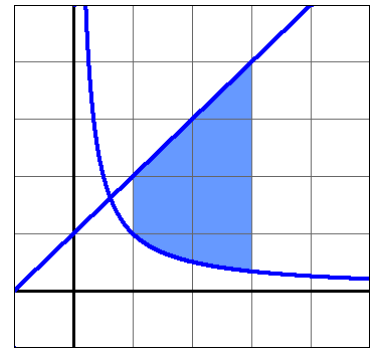


In earlier Lectures, as long as y -coordinates on an interval I were positive, we were able to equate the area under a curve on an interval $a \leq x \leq b$ with the definite integral from a to b : $\int_a^b f(x) dx$. Combining this concept with geometric ideas such as those used in Example A give us a means of finding the area of a region located between curves.

Example B: Find the area between the curves $f(x) = x + 1$ and $g(x) = \frac{1}{x}$ on the interval $1 \leq x \leq 3$.

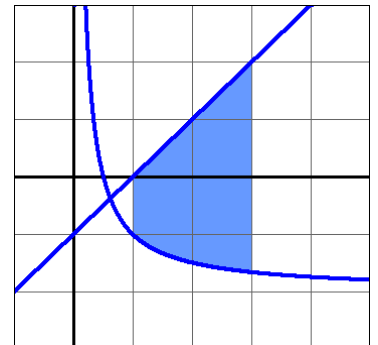
As long as $f(x) > g(x)$ for all x in the interval, we can use the subtraction property to find the area.

answer: $6 - \ln 3$



Example C: Find the area between the curves $f(x) = x - 1$ and $g(x) = \frac{1}{x} - 2$ on the interval $1 \leq x \leq 3$.

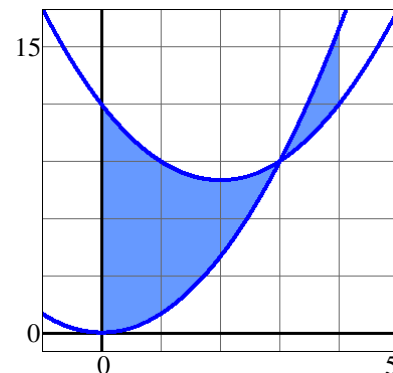
The shape remains the same because the curves from Example B were simply shifted down by 2. The area should be the same. However, in this case some of the area lies below the x -axis, and would thus be a negative amount (as noted in earlier Lectures). Does this affect the calculation of area between the two curves?



So the integral calculation gives us the same result as in Example B: $\text{area} = 6 - \ln 3$. It doesn't matter whether the two curves lie above or below the x -axis, only that we subtract the "higher" curve minus the "lower".

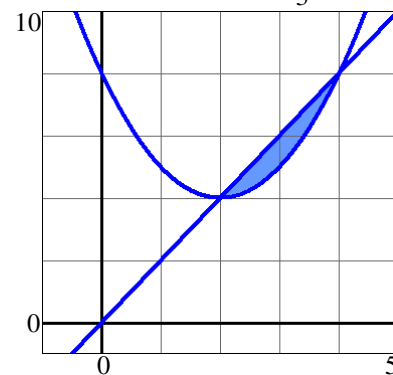
Example D: Find the area between the curves $y = x^2$ and $y = x^2 - 4x + 12$ on the interval $0 \leq x \leq 4$.

Looking at the graph (window $[-1, 5]$ by $[-1, 17]$), $y = x^2 - 4x + 12$ lies above $y = x^2$ for a portion of the interval, but is below for the rest of the interval. We need to determine where the two intersect, then set up two integrals: one for each portion. *answer: 20*

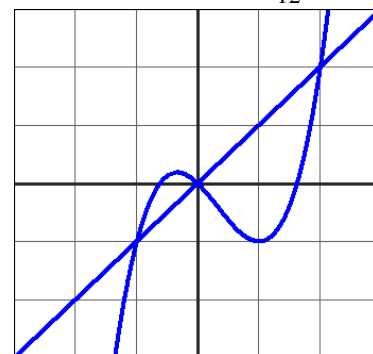


Example E: Find the area between the curves $y = 2x$ and $y = x^2 - 4x + 8$.

In this case, although no interval is specified, by looking at the graph (window $[-1, 5]$ by $[-1, 10]$) we can see that there are two points of intersection. Between these two points the area is bounded, and lies between the two curves. We need to solve for the intersections to find the boundaries (or limits) of integration. *answer: $\frac{4}{3}$*



4.9 Example A revisited: Find the area between the curves $f(x) = x^3 - x^2 - x$ and $g(x) = x$. *answer: $\frac{37}{12}$*



The key to all of these is looking at the graph to determine which function is above the other, finding intersections as needed, then setting up and evaluating the appropriate integrals.

Example F: Find the area between the curves $x = y^2$ and $y = x - 2$. *answer:* $\frac{9}{2}$

