15

Statistics

15.1 Formulating Questions, Designing Investigations, and Gathering Data

Class Activity 15A: Challenges in Formulating Survey Questions

Lincoln Elementary School would like to add soup to its school lunch menu. The school staff decides to poll the students to learn about the soups they like. Compare the questions in 1 through 5. How might the data that the school would collect be different for the different questions? What are the advantages and disadvantages of each question?

1. What is your favorite soup?
2. If you had to pick a soup to eat right now, what soup would you pick?
3. Vote for one of the following soups you like to eat:
   - tomato
   - chicken noodle
   - vegetable
   - other
4. Circle all of the following soups you like to eat:
   - tomato
   - chicken noodle
   - vegetable
5. (Write your own question.)

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Class Activity 15B: Choosing a Sample

A college newspaper wants to find out how the students at the college would answer a specific question of importance to the student body. There are too many students for the newspaper staff to ask them all. So the staff decides to choose a sample of students to ask. For each of the following ways that the newspaper staff could select a sample, discuss whether the sample is likely to be representative of the full student body or if there are reasons why the sample may not be representative.

a. Ask their friends.

b. Ask as many of their classmates as they can.

c. Stand outside the buildings their classes are in and ask as many people as they can who come by.

d. Stand outside the student union or other common meeting area, and try to pick people who they think are representative of the students at their institution to ask the question.

e. Generate a list of random numbers between 1 and the number of students at the college. (Many calculators can generate random numbers; random numbers can also be generated on the Internet; go to *www.pearsonhighered.com/beckmann*.) Pick names out of the student phone book corresponding to the random numbers (e.g., for 123, pick the 123rd name), and contact that person by phone or by e-mail.

Class Activity 15C: Using Random Samples

1. At a factory that produces computer chips, a batch of 5000 computer chips has just been produced. To check the quality of the computer chips, a random sample of 100 computer chips is selected to test for defects. Of these 100 chips, 3 were

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found to be defective. Based on these results, what is the best estimate you can give for the number of defective computer chips in the batch of 5000? Find several different ways to solve this problem, including ways that elementary school children might be able to develop.

2. Mr. Lawler had a bag filled with 160 plastic squares. The squares were identical, except that some were yellow and the rest were green. A student in Mr. Lawler's class randomly picked out 20 of the squares; 4 of the squares were yellow and the rest were green. Mr. Lawler asked his students to use this information to make their best scientific estimate for the total number of yellow squares that were in the bag. Mr. Lawler's students had several different ideas. For each of the following initial ideas, discuss the idea and describe how to use it to estimate the total number of yellow squares. Which ideas are related?

a.

20	20	20	20	20	20	20	20
↓	↓	↓	↓	↓	↓	↓	↓
4	4	4	4	4	4	4	4

b.

20	→	4
40	→	8
60	→	12
80	→	16
100	→	20
120	→	24
140	→	28
160	→	32

c. Of the 20 squares Taryn picked, $\frac{1}{5}$ were yellow.

yellow				

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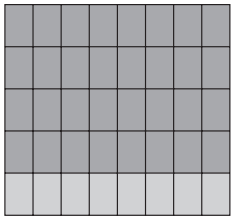
d. Taryn picked $\frac{1}{8}$ of the squares in the bag.

e. $$\frac{4}{20} = \frac{8}{40} = \frac{12}{60} = \frac{16}{80} = \frac{20}{100} = \frac{24}{120} = \frac{28}{140} = \frac{32}{160}$$

f.



g.

$$\frac{4}{20} \overset{\times 8}{=} \frac{x}{160}$$

h.

$$\times 5 \left(\frac{4}{20} = \frac{x}{160} \right) \times 5$$

Class Activity 15D: Using Random Samples to Estimate Population Size by Marking (Capture–Recapture)

You will need a bag filled with a large number (at least 100) of small, identical beans or other small objects that can be marked (such as small paper strips or beads that can be colored with a marker).

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Pretend that the beans are fish in a lake. You will estimate the number of fish in the lake without counting them all by using a method called *capture–recapture.*

1. Go "fishing:" Pick between 20 and 50 "fish" out of your bag. Count the number of fish you caught, and label each fish with a distinctive mark. Then throw your fish back in the lake (the bag) and mix them thoroughly.

2. Go fishing again: Randomly pick about 50 fish out of your bag. Count the total number of fish you caught this time, and count how many of the fish are marked.

3. Use your counts from parts 1 and 2 to estimate the number of fish in your bag. Explain your reasoning.

4. When Ms. Wade used the method described in parts 1 through 3, she picked 30 fish at first, marked them, and put them back in the bag. Ms. Wade thoroughly mixed the fish in the bag and randomly picked out 40 fish. Of these 40 fish, 5 were marked. The children in Ms. Wade's class had several different ideas for how to determine the total number of fish in the bag. For each of the following initial ideas, discuss the idea and describe how to use the idea to determine approximately how many fish are in the bag. Which ideas are related?

a.

40	40	40	40	40	40
↓	↓	↓	↓	↓	↓
5	5	5	5	5	5

b.

40	→	5
80	→	10
120	→	15
160	→	20
200	→	25
240	→	30

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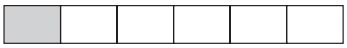
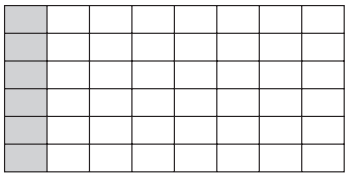
c. $\frac{1}{8}$ of the fish are marked.

d. $\frac{1}{6}$ of the marked fish were chosen.



e. $\frac{5}{40} = \frac{10}{80} = \frac{15}{120} = \frac{20}{160} = \frac{25}{200} = \frac{30}{240}$

f.



g.

$$\overset{\times 6}{\frown}\quad \frac{5}{40} = \frac{30}{x} \quad \overset{\times 6}{\smile}$$

h.

$$\times 8 \left(\frac{5}{40} = \frac{30}{x} \right) \times 8$$

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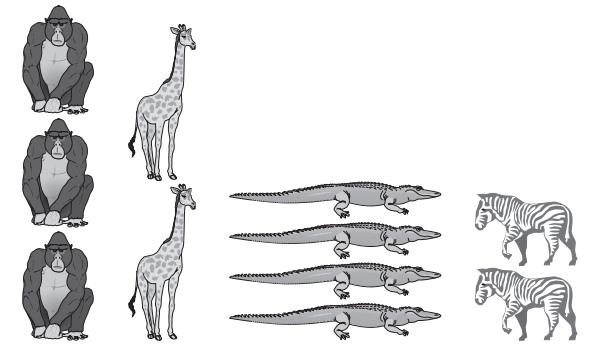
15.2 Displaying Data and Interpreting Data Displays

Class Activity 15E:

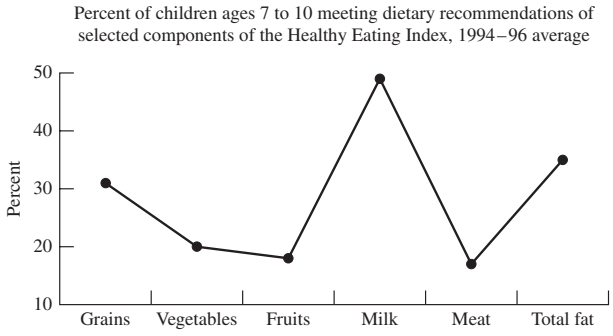
What Is Wrong with These Displays?

1. Ryan scooped some small plastic animals out of a tub, sorted them, and made a pictograph like the following:



What is a problem with Ryan's pictograph?

2. What is wrong with the next data display? Show how to fix it.



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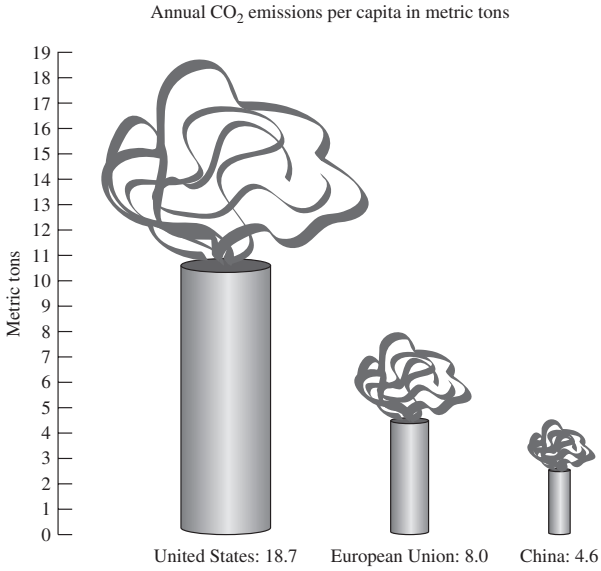
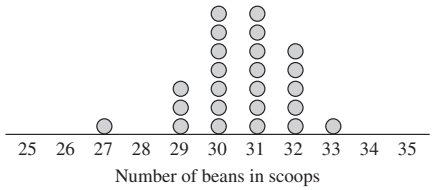
3. What is misleading about the next display of annual per capita carbon dioxide emissions in various countries? *Hints*: Think back to Section 14.5 on how scaling affects volume.



Class Activity 15F: What Is Wrong with the Interpretation of These Displays?

1. Students scooped dried beans out of a bag and counted the number of beans in the scoop. Each time, the number of beans in the scoop was recorded in the next dot plot.



When a student was asked to make a list of the data that are displayed in the dot plot, the student responded thus:

1, 3, 7, 7, 5, 1

What is wrong with the student's response? What is a correct list of the data that are displayed in the dot plot? What do the student's numbers tell us?

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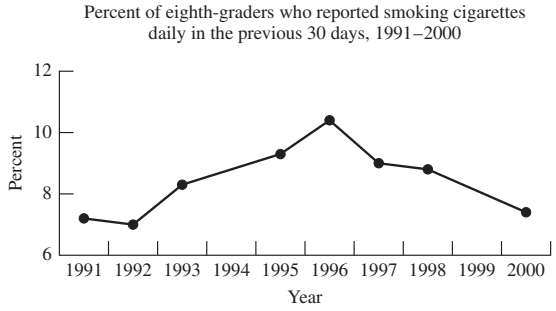
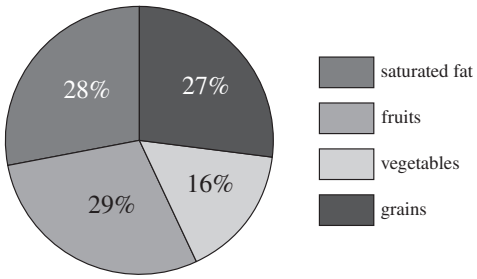
2. Consider the next line graph about adolescents' smoking. Based on this display, would it be correct to say that the percentage of eighth-graders who reported smoking cigarettes daily in the previous 30 days was about twice as high in 1996 as it was in 1993? Why or why not?



3. Consider the next table on children's eating habits.

Food	Percent of 4–6-year-olds meeting the dietary recommendation for the food
Grains	27%
Vegetables	16%
Fruits	29%
Saturated fat	28%

Would it be appropriate to use a single pie graph, as shown here, to display this information? Explain your answer.



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Class Activity 15G: Three Levels of Questions about Graphs

Recall that the three levels of graph comprehension discussed in the text are as follows:

Reading the data. This level of comprehension requires a literal reading of the graph. The reader simply "lifts" the facts explicitly stated in the graph, or the information found in the graph title and axes labels, directly from the graph. There is no interpretation at this level.

Reading between the data. This level of comprehension includes the interpretation and integration of the data in the graph. It requires the ability to compare quantities (e.g., greater than, tallest, smallest) and the use of other mathematical concepts and skills (e.g., addition, subtraction, multiplication, division) that allow the reader to combine and integrate data and identify the mathematical relationships expressed in the graph.

Reading beyond the data. This level of comprehension requires the reader to predict or infer from the data by tapping existing knowledge and knowledge developed from "reading the data" and "reading between the data" for information that is neither explicitly nor implicitly stated in the graph.

The following examples are questions at the different graph-reading levels. All questions are about a bar graph that shows the heights of children in a class ([3, p. 35]):

- What would be a good title for this graph? (Read between the data)
- How tall is (insert a name)? (Read the data)
- Who is the tallest of the students on the graph? (Read between the data)
- Who do you think is the oldest? Why? Can this be answered directly from the graph? (Read beyond the data)
- Who do you think has the smallest shoe size? Why? Can this be answered directly from the graph? (Read beyond the data)
- Who do you think weighs the least? Why? (Read beyond the data)

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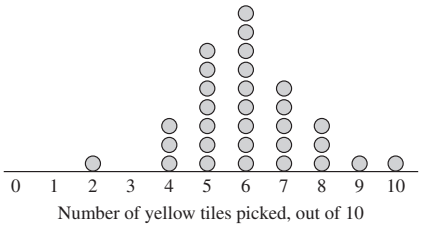
1. In your classroom you have a box with 100 small square tiles in it. The tiles are identical except that some are yellow and the rest are blue. Your students take turns picking 10 tiles out of the box without looking. Then they record the number of yellow squares (out of the 10) they picked on a sticky note and put the tiles back in the box. The class uses the sticky notes to make a dot plot on the chalkboard. It winds up looking like the one shown next (where each dot represents a sticky note).



a. Write two "read the data" questions for the dot plot. Answer your questions.

b. Write two "read between the data" questions for the dot plot. Answer your questions.

c. Write two "read beyond the data" questions for the dot plot. Answer your questions (to the extent possible).

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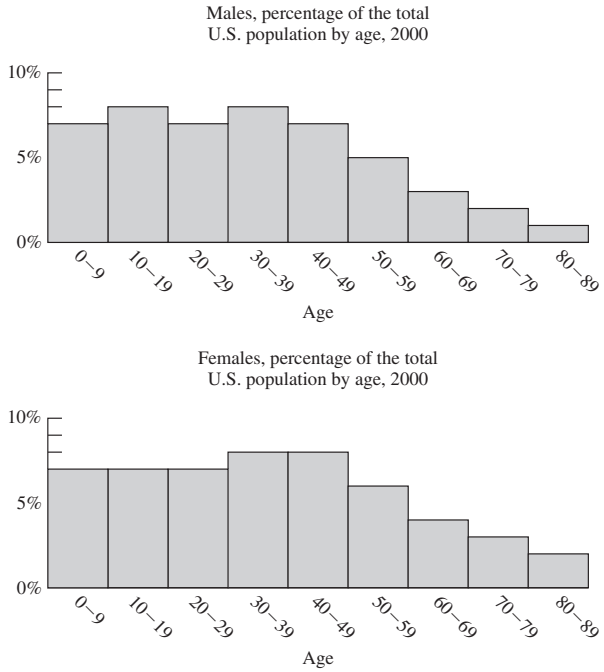
2. Consider the next two histograms, which are based on U.S. Census Bureau data from the 2000 census.



Write "read the data," "read between the data," and "read beyond the data" questions for the histograms. Questions may be suitable to ask separately about each histogram or may be about the two histograms together and how they compare. Answer your questions (to the extent possible).

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Class Activity 15H: Display These Data about Pets

A class collected information about the pets they have at home, as shown in the next table.

Name	Pets at Home
Michelle	1 dog, 2 cats
Tyler	1 salamander, 2 snakes, 3 dogs
Antrice	hamster
Yoon-He	cat
Anne	none
Peter	2 dogs
Brandon	guinea pig
Brittany	1 dog, 1 cat

Name	Pets at Home
Orlando	none
Chelsey	2 dogs, 10 fish
Sarah	1 rabbit
Adam	none
Lauren	2 dogs
Letitia	3 cats
Jarvis	1 dog

1. Consider the following questions about pets:

- **a.** Are dogs our most popular pet?
- **b.** How many pets do most people have?
- **c.** How many people have more than one pet?
- **d.** Are most of our pets mammals?
- **e.** Write some other questions about their pets that may be of interest to students and that could be addressed by the data that were collected.

2. Make each of the following data displays and use them to answer the questions from part 1. Observe that different graphs will be helpful for answering different questions.

- **a.** A dot plot that shows how many students have 0 pets, 1 pet, 2 pets, 3 pets, and so on
- **b.** A bar graph that shows how the *students* in the class fall into categories depending on what kind of pet they have
- **c.** Another bar graph like the one in part (b), except pick the categories in a different way this time
- **d.** A bar graph that shows how the *pets* of students in the class fall into categories.

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Class Activity 15I: Investigating Small Bags of Candies

For this activity, each person, pair, or small group in the class needs a small bag of multi-colored candies. All bags should be of the same size and consist of the same type of candy. Bags should not be opened until after completion of the first part of this activity.

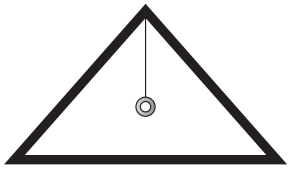
1. Do not open your bag of candy yet! Write a list of questions that the class as a whole could investigate by gathering and displaying data about the candies.

2. Open your bag of candy (but do not eat it yet!) and display data about your candies in two significantly different ways. For each display, write and answer questions at the three different graph-reading levels.

3. Together with the whole class, collect and display data about the bags of candies in order to answer some of the questions the class posed in part 1.

Class Activity 15J: The Length of a Pendulum and the Time It Takes to Swing

A fifth-grader's science fair project[1] investigated the relationship between the length of a pendulum and the time it takes the pendulum to swing back and forth. The student made a pendulum by tying a heavy washer to a string and attaching the string to the top of a triangular frame, as pictured. The



[1]Thanks to Arianna Kazez for the data and information about the project.

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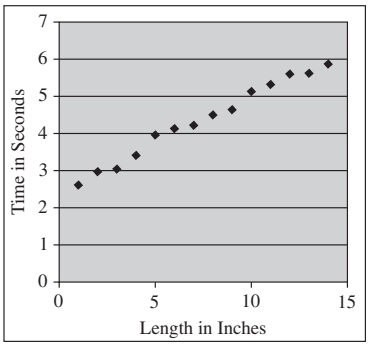
length of the string could be varied. The next table and scatterplot show how long it took the pendulum to swing back and forth 10 times for various lengths of the string. (Several measurements were taken and averaged.)

length of string in inches	time of 10 swings in seconds
1	2.61
2	2.97
3	3.04
4	3.41
5	3.96
6	4.13
7	4.22
8	4.5
9	4.64
10	5.13
11	5.32
12	5.56
13	5.62
14	5.87



1. Write one or two questions about the scatterplot for each of the three graph-reading levels. Answer each question (to the extent possible).

2. Arianna used her science fair data to predict how long it would take a pendulum with 100 inches of string to swing back and forth 10 times. She started by observing that for every 4 inches of string it takes 1 second longer. Explain how to use this observation to determine approximately how long it might take a 100-inch pendulum to swing back and forth 10 times.*

3. Arianna used her science fair data to predict how long a string she would need so that one swing would take 1 second (like a grandfather clock). She started by observing that for every 4 inches of string it takes 1 second longer. She also used the fact that a 14-inch pendulum took 5.87 seconds for 10 swings. Arianna knew she needed to get to 10 seconds. Explain how to use these ideas to determine approximately how long a string is needed so that one swing will take 1 second.*

*This provides a good initial estimate, but according to physical theories, the estimate won't be fully correct.

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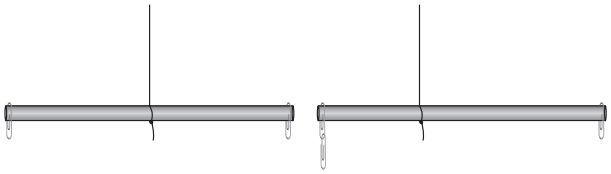
Class Activity 15K: Balancing a Mobile

For this activity, each person, pair, or small group in the class needs a drinking straw, string, tape, at least 7 paper clips of the same size, a ruler, and graph paper. Participants will use the straw, string, tape, and paper clips to make a simple mobile.



1. Tie one end of the string snugly around the straw. Tape one paper clip to each end of the straw. Hold the other end of the string so that your mobile hangs freely. Adjust the location of the string along the straw so that the straw balances horizontally. The string should now be centered on the straw, as in the picture, on the left. Measure the distance on the straw from the string to each end.
2. Repeatedly add one more paper clip to one side of the straw (but not to the other side). Every time you add a paper clip, adjust the string so that the straw balances horizontally. Each time, measure the distance on the straw from the string to the end that has multiple paper clips, and record your data.
3. Make a graphical display of your data from part 2. (Use graph paper.)
4. Write and answer several questions at each of the three different graph-reading levels about your graphical display in part 3.