Calculus 241, section 11.6 Planes in Space

notes by Tim Pilachowski

We have already noted that any three distinct points lie in the same plane.
Likewise, any two distinct (i.e. non-parallel) vectors will lie in the same plane. (We'll think of the two vectors as sharing the same initial point.)
Now do we turn this information into the equation of a plane in 3-D space?

Consider an alternate approach. "There is only one plane that contains a given point and is perpendicular to a given line." Considering that a line will be parallel to a given nonzero vector, we can use a perpendicular (which we know how to find) to that vector and the vector itself to determine the equation of a plane.

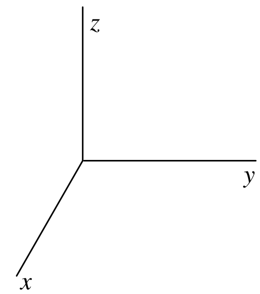
We'll say that the perpendicular vector is **normal** to a plane $\mathcal{P}$, and designate it by **N** or $\vec{N}$.
Then, given two points on plane $\mathcal{P}$, $P_0 = (x_0, y_0, z_0)$ and $P = (x, y, z)$, and a normal to the plane, $\vec{N} = a\vec{i} + b\vec{j} + c\vec{k}$, then it must be true that $\vec{N} \bullet \overrightarrow{P_0P} = 0$ [section 11.3 Corollary 11.7a].

$$\vec{N} \bullet \overrightarrow{P_0P} = 0$$
$$\left(a\vec{i} + b\vec{j} + c\vec{k}\right) \bullet \left((x - x_0)\vec{i} + (y - y_0)\vec{j} + (z - z_0)\vec{k}\right) = 0$$
$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$
$$ax + by + cz = ax_0 + by_0 + cz_0$$
$$ax + by + cz = d$$

Observations:

Example A. Find an equation of the plane containing the points $P = (3, 2, 1)$, $Q = (7, 2, 3)$, and $R = (-6, 4, -2)$.

Example B: Graph the first-octant portion of the plane $4x + 2y + 3z = 12$.



Finding the distance from a point to a plane is, as your text notes, greatly simplified by using vector methods.

Theorem 11.13: Let $\mathcal{P}$ be a plane with normal $\mathbf{N}$, let P_1 be any point not on $\mathcal{P}$, and let P_0 be any point on $\mathcal{P}$. Then the distance D between P_1 and $\mathcal{P}$ is given by

Example A extended. Find the distance between the point (1, 1, 1) and the plane containing the points $P = (3, 2, 1)$, $Q = (7, 2, 3)$, and $R = (-6, 4, -2)$.