Calculus 241, section 14.9 Parametrized Surfaces

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Back in section 6.7, the topic was curves parametrized by $x = f(t)$ and $x = g(t)$.

In section 12.1, the topic was vector-valued functions, $\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$, where the component functions could be expressed parametrically. Recall that the sketch of a curve C traced out by a vector-valued function was in the form of coordinate points (x, y, z) whose values are determined by the component functions, i.e. the terminal points of the vectors that were the range of $\vec{r}$.

Now, in section 14.9, we'll do much the same for surfaces Σ traced out by a vector-valued function $\vec{r}(u, v) = x(u, v)\vec{i} + y(u, v)\vec{j} + z(u, v)\vec{k}$. As with the curves in section 12.1, we'll be thinking not of the vectors themselves, but rather the coordinate points (x, y, z) whose values are the terminal points of the vectors that are the range of $\vec{r}$.

The process is another "change of variables" situation. (When we involve integration in chapter 15, we'll be seeing the Jacobian again.)

Note that, as always, the choice of letters u and v is somewhat arbitrary, and serve as the generic version for definition purposes. For Examples and for homework practice, we'll be seeing $\vec{r}(x, y)$, $\vec{r}(r, \theta)$, $\vec{r}(z, \theta)$ and $\vec{r}(\phi, \theta)$.

Example A: Find a parametrization of the surface Σ which is the portion of the plane $z = 2$ for which $0 \le x \le 3$ and $1 \le y \le 5$.

Example B: Find a parametrization of the surface Σ which is the part of the plane $x + 2y + 3z = 30$ to the right of the rectangle in the xz-plane with opposite corners (0, 0, 0) and (5, 0, 7).

Example C: Find a parametrization of the surface Σ which is the portion of the plane $z = 2$ inside $r = 3$.

Example D: Find a parametrization of the surface Σ which is the cylinder $x^2 + (y-3)^2 = 9$.

Example E: Find a parametrization of the surface Σ which is the portion of the plane $y = -2$ inside the cylinder $x^2 + z^2 = 4$.

Example F: Find a parametrization of the surface Σ which is the portion of the cylinder $y^2 + z^2 = 4$ between the planes $x = 1$ and $x = 5$.

Example G: Find a parametrization of the surface Σ which is the portion of the cone $z = \sqrt{x^2 + y^2}$ for which $0 \le z \le 3$.

Example H: Find a parametrization of the surface Σ which is the sphere $x^2 + y^2 + z^2 = 9$.

Example I: Find a parametrization of the surface Σ which is the portion of the sphere $x^2 + y^2 + z^2 = 9$ inside the cone $\phi = \dfrac{\pi}{4}$.