Math 241 Sections 01 Exam 1 Sample 4 Solutions** **Dr. Justin O. Wyss-Gallifent**

1. Given the following data:

$$P = (-1, 0, 3)$$
$$Q = (2, 5, 5)$$
$$\bar{a} = 1\,\hat{\imath} + 2\,\hat{\jmath} + 0\,\hat{k}$$
$$\bar{b} = 3\,\hat{\imath} + 2\,\hat{\jmath} + 1\,\hat{k}$$

   (a) We use:

$$2\left(\frac{\overrightarrow{PQ}}{||\overrightarrow{PQ}||}\right) = 2\left(\frac{3\,\hat{\imath} + 5\,\hat{\jmath} + 2\,\hat{k}}{\sqrt{9+25+4}}\right)$$

   (b) We have:

$$\begin{aligned} \bar{a}\cdot\bar{b} &= ||\bar{a}||||\bar{b}||\cos\theta \\ 7 &= \sqrt{1+4+0}\sqrt{9+4+1}\cos\theta \\ \cos\theta &= \frac{7}{\sqrt{5}\sqrt{14}} \end{aligned}$$

   (c) We have:

$$\begin{aligned} \mathrm{Pr}_{\bar{a}}\bar{b} &= \left(\frac{\bar{a}\cdot\bar{b}}{\bar{a}\cdot\bar{a}}\right)\bar{a} \\ &= \left(\frac{7}{5}\right)(1\,\hat{\imath} + 2\,\hat{\jmath} + 0\,\hat{k}) \end{aligned}$$

2. (a) The line has $\bar{L} = 3\,\hat{\imath} + 0\,\hat{\jmath} + 1\,\hat{k}$ and $P = (-2,-2,3)$. We have $Q = (3,2,1)$ off the line. Therefore

$$\overrightarrow{PQ} = 5\,\hat{\imath} + 4\,\hat{\jmath} - 2\,\hat{k}$$

and so the distance is:

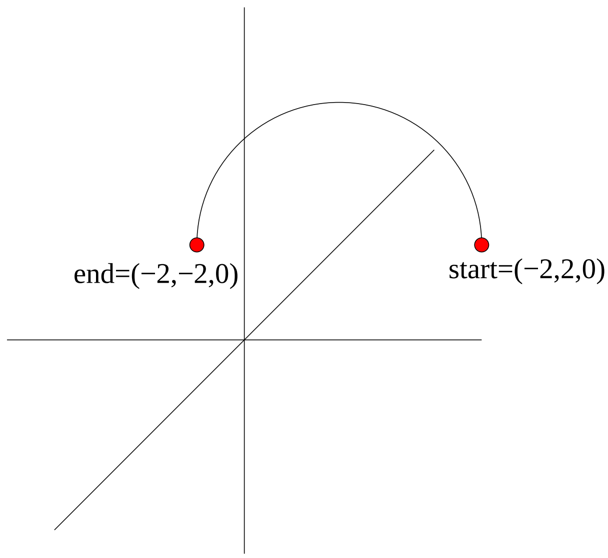
$$\begin{aligned}
\text{Dist} &= \frac{||\bar{L} \times \overrightarrow{PQ}||}{||\bar{L}||} \\
&= \frac{||-4\,\hat{\imath} + 11\,\hat{\jmath} + 12\,\hat{k}||}{\sqrt{9+1+0}} \\
&= \frac{\sqrt{16+121+144}}{\sqrt{9+1+0}}
\end{aligned}$$

(b) We have

$$\bar{r}'(t) = -\sin(t)\,\hat{\imath} + 3\cos(t)\,\hat{\jmath}$$

$$\begin{aligned}
\bar{T}(\pi/4) &= \frac{\bar{r}'(\pi/4)}{||\bar{r}'(\pi/4)||} \\
&= \frac{-\sin(\pi/4)\,\hat{\imath} + 3\cos(\pi/4)\,\hat{\jmath}}{||-\sin(\pi/4)\,\hat{\imath} + 3\cos(\pi/4)\,\hat{\jmath}||} \\
&= \frac{-\frac{\sqrt{2}}{2}\,\hat{\imath} + \frac{3\sqrt{2}}{2}\,\hat{\jmath}}{||-\frac{\sqrt{2}}{2}\,\hat{\imath} + \frac{3\sqrt{2}}{2}\,\hat{\jmath}||} \\
&= \frac{-\frac{\sqrt{2}}{2}\,\hat{\imath} + \frac{3\sqrt{2}}{2}\,\hat{\jmath}}{\sqrt{\frac{1}{2}+\frac{9}{2}}} \\
&= \frac{-\frac{\sqrt{2}}{2}\,\hat{\imath} + \frac{3\sqrt{2}}{2}\,\hat{\jmath}}{\sqrt{5}} \\
&= \frac{\sqrt{10}}{10}\,\hat{\imath} + \frac{3\sqrt{10}}{10}\,\hat{\jmath}
\end{aligned}$$

3. (a) We have



(b) We have three pieces:

$$\bar{r}(t) = t\,\hat{\imath} + (4 - t^2)\,\hat{\jmath} \quad \text{with } 0 \leq t \leq 2$$
$$\bar{r}(t) = (2 - t)\,\hat{\imath} + 0\,\hat{\jmath} \quad \text{with } 0 \leq t \leq 2$$
$$\bar{r}(t) = 0\,\hat{\imath} + t\,\hat{\jmath} \quad \text{with } 0 \leq t \leq 4$$

4. (a) If it's perpendicular to both then we can use the cross product so

$$\bar{L} = (2\,\hat{\imath} + 2\,\hat{\jmath} + 0\,\hat{k}) \times (3\,\hat{\imath} - 1\,\hat{\jmath} + 2\,\hat{k}) = 4\,\hat{\imath} - 4\,\hat{\jmath} - 8\,\hat{k}$$

and so we have

$$\begin{aligned} x &= 1 + 4t \\ y &= 2 - 4t \\ z &= 3 - 8t \end{aligned}$$

(b) The line hits the plane when

$$\begin{aligned} 2(2t + 2) + (5 - t) - (t + 10) &= 1 \\ 4t + 4 + 5 - t - t - 10 &= 1 \\ 2t &= 2 \\ t &= 1 \end{aligned}$$

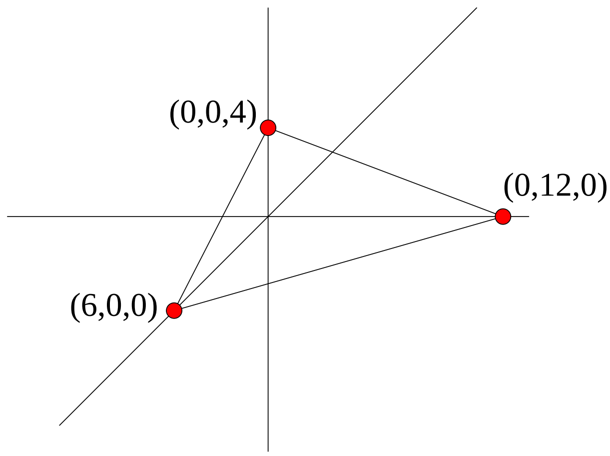
and this occurs at the point given by

$$\bar{r}(1) = 4\,\hat{\imath} + 4\,\hat{\jmath} + 11\,\hat{k}$$

which is

$$(4, 4, 11)$$

5. (a) We have



(b) We have

$$\bar{r}'(t) = 2t\,\hat{\imath} + 3t^2\,\hat{\jmath} + 0\,\hat{k}$$

and so the length is

$$\int_0^1 ||\bar{r}'(t)||\ dt = \int_0^1 \sqrt{4t^2 + 9t^4 + 0}\ dt$$