Math 241 Exam 2 Sample 2

Directions: Do not simplify unless indicated. No calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem.

Please put problem 1 on answer sheet 1

1. (a) Let $f(x,y) = 3x^2 - 2xy - y$. Suppose the unit vector $\bar{u}$ points at an angle of $60°$ from $\nabla f(x,y)$. Determine $D_{\bar{u}} f(2,1)$. [5 pts]

   (b) Suppose $z = 2x^2 + xy - xy^2$, $x = 2s + t$ and $y = e^t$. Find $\frac{\partial z}{\partial t}$ in terms of s and t. [6 pts]

   (c) Let C be the curve $x^2 + 2xy - 3y^2 = -10$. Find all points on C where C is parallel to $2\,\hat{\imath} + \hat{\jmath}$. Hint: Look at ∇f for an appropriate f. [9 pts]

Please put problem 2 on answer sheet 2

2. (a) Let $f(x,y) = x - y^2$. On a single Cartesian plane draw the level curves $f(x,y) = c$ for $c = -2, -1, 0, 1, 2$. Label each curve with its value of c. [10 pts]

   (b) Find the equation of the plane tangent to the elliptical cylinder $x^2 + 2z^2 = 22$ at $(2, 7, -3)$. [10 pts]

Please put problem 3 on answer sheet 3

3. (a) Sketch the surface $z = 4 - x^2$. Be sure to include some points or tick marks to give a sense of scale/position. [5 pts]

   (b) Sketch the surface $\frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1$. Be sure to include some points or tick marks to give a sense of scale/position. [5 pts]

   (c) Write down the equation for a cone opening down with vertex at $(0, 0, 5)$. [5 pts]

   (d) Write down the equation for the vertical plane passing through the two points $(3, 0, 0)$ and $(0, 2, 0)$. [5 pts]

Please put problem 4 on answer sheet 4

4. Find all critical points for $f(x,y) = x^2y - 2xy + 2y^2 - 15y$ and use the second derivative test to determine whether each is a relative maximum, relative minimum or saddle point. [20 pts]

Please put problem 5 on answer sheet 5

5. Assuming a, b and r are fixed positive constants, use Lagrange Multipliers to find the maximum and minimum of $f(x,y) = ax + by$ on the circle of radius r centered at the origin. [20 pts]

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