

### Math 241 Exam 3 Sample 1

**Directions:** Do not simplify or evaluated unless indicated. No calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem.

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#### Please put problem 1 on answer sheet 1

- (a) Let  $R$  be the region inside  $r = 1 + \sin \theta$ , above  $y = \frac{3}{4}$  and in the first quadrant. [10 pts]  
Draw a picture of  $R$ . Set up an iterated integral in polar coordinates for  $\iint_R x^2 y \, dA$ .
- (b) Let  $D$  be the solid in the first octant, inside the sphere  $x^2 + y^2 + z^2 = 9$  and below the cone  $z = \sqrt{3x^2 + 3y^2}$ . Set up an iterated integral in spherical coordinates for  $\iiint_D z \, dV$ . [10 pts]

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#### Please put problem 2 on answer sheet 2

- (a) Let  $R$  be the region in the first quadrant bounded by the curve  $y = 9 - x^2$ . Draw a picture of  $R$  and then set up an iterated integral for  $\iint_R x \, dA$  with  $R$  as HS. [10 pts]
- (b) Let  $R$  be the region in the second quadrant and bounded by the line  $y = x + 3$ . Draw a picture of  $R$  and then set up an iterated integral for  $\iint_R x^2 + y^2 \, dA$  with  $R$  as polar. [10 pts]

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#### Please put problem 3 on answer sheet 3

- (a) Parametrize the cylinder of radius 2 centered along the  $x$ -axis with  $-1 \leq x \leq 3$ . [5 pts]
- (b) Draw the surface  $r(x, z) = x \hat{i} + 2 \hat{j} + z \hat{k}$  for  $0 \leq x \leq 3$  and  $0 \leq z \leq 4$ . [5 pts]
- (c) Evaluate the following integral. [10 pts]

$$\int_0^2 \int_x^2 \sin(y^2) \, dy \, dx$$

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#### Please put problem 4 on answer sheet 4

- (a) Let  $D$  be the solid inside the cylinder  $r = \sin \theta$ , above  $z = 0$  and below  $z = 9 - x^2 - y^2$ . [10 pts]  
Draw separate pictures of  $D$  and its corresponding  $R$  and then set up an iterated integral in cylindrical coordinates for  $\iiint_D y \, dV$ .
- (b) Evaluate the integral [10 pts]

$$\int_0^1 \int_x^0 \int_0^{x+y} y \, dz \, dy \, dx$$

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#### Please put problem 5 on answer sheet 5

- Perform a change of variables to rewrite the integral  $\iint_R x \, dA$ , where  $R$  is the region bounded by the lines  $y = \frac{1}{x}$ ,  $y = \frac{2}{x}$ ,  $y = x$  and  $y = 4x$ . Draw a picture of  $R$ , your new region  $S$  and write the new integral as an iterated double integral in  $u$  and  $v$ . [20 pts]

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The End!