Math 241 Exam 4 Sample 1

Directions: Carefully read the directions as to whether to evaluate your integrals. No calculators are permitted. Show all work as appropriate for the methods taught in this course. Partial credit will be given for any work, words or ideas which are relevant to the problem.

Please put problem 1 on answer sheet 1

1. Let Σ be the portion of $y = 4 - x^2$ with $y \geq 0$ and between $z = 1$ and $z = 3$, oriented to the left. Draw a picture of Σ and find the rate at which the fluid $\bar{F}(x, y, z) = y\,\hat{\jmath}$ is flowing through Σ. [20 pts]
   Stop when you have an iterated double integral.

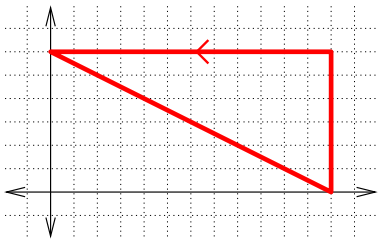
Please put problem 2 on answer sheet 2

2. (a) Evaluate $\int_C y\ dx + (x+1)\ dy$ where C is an unknown curve from $(2, -5)$ to $(7, 1)$. [5 pts]

   (b) Find the mass of the wire C, where C is the quarter-ellipse $4x^2 + y^2 = 4$ in the first quadrant and the density is $\delta(x, y) = 3xy$. [15 pts]

Please put problem 3 on answer sheet 3

3. Evaluate $\int_C 5x\ dx + \frac{1}{2}x^2\ dy$ where C is the oriented curve shown in the picture. [20 pts]



Please put problem 4 on answer sheet 4

4. Let Σ be the portion of the paraboloid $z = 9 - x^2 - y^2$ in the first octant. Let C be the edge of Σ with counterclockwise orientation when viewed from above. Draw separate pictures of Σ and of C with their orientations. Use Stokes' Theorem to find the work done on a particle by the force $\bar{F}(x, y, z) = x\,\hat{\imath} + yz\,\hat{\jmath} + xz\,\hat{k}$ as the particle traverses the curve C. [20 pts]
   Stop when you have an iterated double integral.

Please put problem 5 on answer sheet 5

5. (a) Let Σ be the hemisphere $x^2 + y^2 + z^2 = 4$ above the xy-plane along with the disk $x^2 + y^2 \leq 4$ in the xy-plane. Find the rate at which the fluid $\bar{F}(x, y, z) = 2x\,\hat{\imath} + y\,\hat{\jmath} + 7z\,\hat{k}$ is flowing inwards through Σ. [15 pts]

   (b) Explain why (a) would be harder if Σ did not include the disk. [5 pts]

The End!