Stat 400, section 2.1 Experiments, Sample Spaces, and Events

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We begin with probability, the realm of "What could happen?" and "How likely is it to happen?"

Along with counting the number of outcomes, we'll be specifying the nature of those outcomes and (eventually) working out whether a particular outcome has a high or low chance of occurring.

Vocabulary:
For a given situation, or **experiment**, observations are made and data is recorded.

Note that "experiment" in statistics and probability includes science done in a laboratory, but is larger than that as well, and includes social sciences (e.g. psychology, education) along with more informal settings (e.g. "How likely is it that I'll get an A on the first Exam, and what can I do to increase my chances?")

A **sample space** S must contain *all possible* outcomes for an experiment.

A sample space is a set. The elements in a sample space are the outcomes of the experiment.

Using a concept from set theory, the sample space is the universe U for a given experiment.

Elements of a set must be distinct. In this case, we would say that the outcomes in a sample space must be **mutually exclusive**.

An **event** (designated with a capital letter A, B, C, etc.) is a subset of the sample space, and will incorporate one or more of the outcomes.

Before we get done today, we will talk about the **unions**, **intersections** and **complements** of various events. The notations for these will be imported from set theory.

Example A. You toss two coins. Specify the sample space, then define some events from this sample space.

Example A – a new perspective. You toss ten coins and record the number of heads. Specify the sample space then define some events from this sample space.

Example A revisited. You toss a coin until it comes up heads. Specify the sample space then define some events from this sample space.

Example B-1. You toss a standard six-sided die. Specify the sample space then define some events from this sample space.

Find the events $A \cup C$ and $A \cap C$.

Find the events D', $B \cap D^c$, and $B \cup D^c$.

Describe an event F such that E and F are complementary.

List all events G_n such that E and G_n are mutually exclusive.

Identify two events that are equal.

From the Stat 400 page you can link to a supplement, set theory and Venn diagrams, which has a much more detailed explanation of the basics of set theory and how Venn diagrams can be useful in picturing unions, intersections and complements, as well as how they can be used to answer questions about probability.

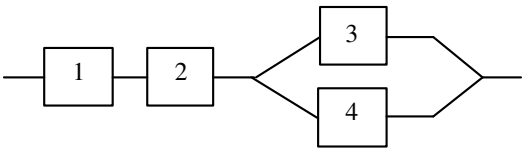
One of the text homework questions has you take a look at DeMorgan's Laws from set theory. They are useful, but not useful enough for this class to take time to memorize.

Example B-2. You toss two standard six-sided dice. Specify the sample space then define some events from this sample space.

Example C. For text and class purposes, you need to be familiar with a standard deck of 52 cards.

Example D. Suppose that a box contains 3 blue blocks and 2 yellow blocks. You pick three blocks without replacement. Specify the sample space then define some events from this sample space.

Example E. Four components are connected to form a system as shown in the diagram below.
Define S = an individual component functions and F = an individual component fails.



Let A = at least 3 components function.

$A =$

Let B = the system as a whole functions.

$B =$

Notes:

Example F. On their menu, Chili's offers "$20 Dinner for Two". Diners are asked to
"SELECT TWO FULL-SIZE ENTREES
•FRIED SHRIMP
•QUESADILLA EXPLOSION SALAD
•CLASSIC BACON BURGER
•CAJUN CHICKEN PASTA".
Describe a sample space of possible outcomes (combinations rather than permutations).

How many outcomes in the sample space correspond to at least one of the diners ordering fried shrimp?

How many outcomes in the sample space correspond to the two diners ordering fried shrimp and the classic bacon burger?

How many outcomes in the sample space correspond to at least one of the diners ordering either fried shrimp or the classic bacon burger?

The examples so far are theoretical situations, the kind used by casinos and lotteries to determine prices and payouts. Often, however, situations are **empirical**, observations made about actual phenomena.

Example G. A hospital records the number of days each ICU patient stays in intensive care. Specify an appropriate sample space.

Each outcome and each event in a sample space will have a **probability** associated with it. We'll investigate more thoroughly in later sections.