

Stat 400, section 2.5 Independence of Events

notes by Tim Pilachowski

In **conditional probability** an outcome or event B is dependent upon another outcome or event A . Formally, $P(B | A) = P(B \text{ given } A)$ = probability that B will happen given the prior condition that A has already happened.

The formal definition and formula for conditional probability is

$$P(B | A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{\text{number of events in } (A \cap B)}{\text{number of events in } (S)}}{\frac{\text{number of events in } (A)}{\text{number of events in } (S)}} = \frac{\text{number of events in } (A \cap B)}{\text{number of events in } (A)} = \frac{N(A \cap B)}{N(A)}$$

In words, when we're considering only event F , what portion/fraction is also part of event E ?

If the presence or absence of the condition (event F) has no effect on the probability of the second event (E), then the two events are independent.

Formally, if two events E and F are independent then

$$P(E | F) = P(E) \text{ and } P(F | E) = P(F).$$

Using the multiplication principle, $P(E \cap F) = P(E) * P(F | E)$. Then, if the two events are independent,

$$P(E \cap F) = P(E) * P(F).$$

Either of these can be used to prove or disprove independence of two events.

A series of events is independent if and only if $P(E_1 \cap E_2 \cap \dots \cap E_n) = P(E_1) * P(E_2) * \dots * P(E_n)$.

Example A-a: Given two events C and D in a sample space S , if we know that $P(C) = 0.3$ and $P(D | C) = 0.2$, then what is $P(C \cap D)$?

Example A-b: Given the same two events C and D as above, if we also know that $P(C \cup D) = 0.64$, are events C and D independent?

Example B: Given two events E and F in a sample space S , if we know that $P(E) = \frac{3}{4}$, $P(F) = \frac{1}{3}$ and

$P(E \cap F) = \frac{1}{4}$, then a) what is $P(E | F)$? b) what is $P(F | E)$? c) Are events E and F independent?

Example C: For two events G and H in a sample space S , we know that they are independent and that $P(G) = 0.7$ and $P(H) = 0.4$.

$$P(G \cap H) =$$

Note:

$$P(G | H) =$$

$$P(H | G^c) =$$

$$P(H^c | G^c) =$$

Example D-1: You pick a card from a standard deck of 52. $N = 52$.

$$C = \text{the card is an Ace} \quad P(C) =$$

$$D = \text{the card is a Spade} \quad P(D) =$$

$$C \cap D =$$

$$P(C \cap D) =$$

$$C | D =$$

$$P(C | D) =$$

$$D | C =$$

$$P(D | C) =$$

Are events C and D independent? How do you know?

Example D-2: You pick two cards from a standard deck of 52.

$$N =$$

$$E = \text{at least one card is an Ace} \quad P(E) =$$

$$E^c = \quad P(E^c) =$$

$$F = \text{picking a pair} \quad P(F) =$$

$$E \cap F =$$

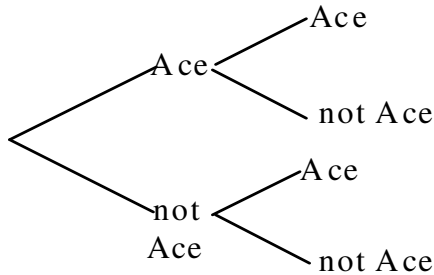
$$P(E \cap F) =$$

Are events E and F independent? How do you know?

From the Stat 400 page you can link to a supplement, [conditional probability and Bayes' Theorem](#), which has an example worked out, then two scenarios with questions designed to help you in all of the concepts and procedures we've covered in chapter 2.

Example D-2 revisited: You pick two cards from a standard deck of 52. (tree diagram method)

E = at least one card is an Ace, F = picking a pair, $E \cap F$ = picking a pair of Aces



a) $P(\text{Ace on first pick}) =$

b) $P(\text{not-Ace on first pick}) =$

c) $P(\text{Ace} | \text{Ace}) =$

d) $P(\text{not-Ace} | \text{Ace}) =$

e) $P(\text{Ace} | \text{not-Ace}) =$

f) $P(\text{not-Ace} | \text{not-Ace}) =$

g) $P(\text{two Aces}) = P(E \cap F) =$

h) $P(\text{Ace then not-Ace}) =$

i) $P(\text{not-Ace then Ace}) =$

j) $P(\text{neither card is an Ace}) =$

k) $P(E) =$

l) $P(F) =$

D-2 answers: a) $\frac{1}{13}$, b) $\frac{12}{13}$, c) $\frac{1}{17}$, d) $\frac{16}{17}$, e) $\frac{4}{51}$, f) $\frac{47}{51}$, g) $\frac{1}{221}$, h) $\frac{16}{221}$, i) $\frac{16}{221}$, j) $\frac{188}{221}$, k) $\frac{33}{221}$, l) $\frac{13}{221}$

Example E: Silver Springs, Florida, has a snack bar and a gift shop. The management observes 100 visitors, and counts 55 who make a purchase in the gift shop (event G), 65 who eat in the snack bar (event H), and 40 who do both.

a) $G | H =$

b) $P(G | H) =$

c) interpretation of $G | H$:

d) $H | G =$

e) $P(H | G) =$

f) interpretation of $H | G$:

g) Are events G and H independent? How do you know?

selected E answers: b) $\frac{8}{13}$, e) $\frac{8}{11}$, f) $\frac{47}{51}$, g) no

Example F: The Gallup organization conducted almost 150,000 interviews from January through May 2009.

	Democrat	Independent	Other	Republican
Female	21%	13%	4%	13%
Male	16%	17%	2%	14%

<http://www.gallup.com/poll/120839/Women-Likely-Democrats-Regardless-Age.aspx>

Results are based on telephone interviews with 149,192 national adults, aged 18 and older, conducted Jan. 2-May 31, 2009, as part of Gallup Poll Daily tracking. Interviews are conducted with respondents on land-line telephones (for respondents with a land-line telephone) and cellular phones (for respondents who are cell-phone only). Categories are arranged in alphabetical order.

Does this table give probabilities of intersections in each cell, or conditional probabilities? How do you know?

Let A = member of the Democratic Party and B = female gender.

a) $P(A) =$ $P(A^c) =$

b) $P(B) =$ $P(B^c) =$

c) $A \cap B =$

d) $P(A \cap B) =$

e) $A \cap B^c =$

f) $P(A \cap B^c) =$

g) $A \cup B =$

h) $P(A \cup B) =$

i) $A^c \cup B =$

j) $P(A^c \cup B) =$

k) $A | B =$ $P(A | B) =$

l) interpretation:

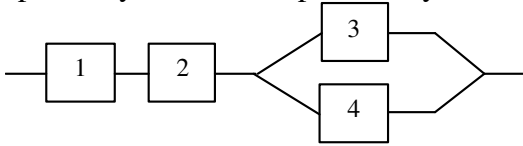
m) $B | A =$ $P(B | A) =$

n) interpretation:

o) Are A and B independent events? How do you know?

selected E answers: a) 0.37, 0.63, b) 0.51, 0.49, d) 0.21, f) 0.16, h) 0.67, j) 0.84, k) 0.41, m) 0.57, o) no

Example G. Four components are connected to form a system as shown in the diagram below. If components work independently one another, and all components have 0.91, 0.92, 0.93 and 0.94 probability of success, respectively, what is the probability that the system will work?



2.4 Example G revisited: Example G: In early 2010, households were surveyed about health insurance coverage, with the following results.

Percentage of persons who lacked health insurance coverage (Table 1), number of persons who lacked health insurance coverage (Table 2), and percentage of persons with public health plan or private health insurance coverage (Table 3), at the time of interview, by age group: United States, Jan–March 2010.

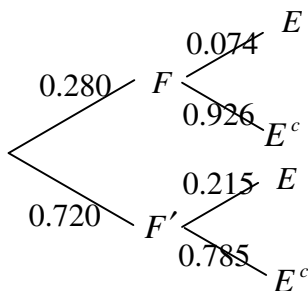
	Under 65 years	18-64 years	Under 18 years
Uninsured %	17.5%	21.5%	7.4%
Number uninsured (millions)	46.4	40.9	5.5
Public %	21.2%	14.4%	38.4%
Private %	62.7%	65.5%	55.5%

Health Insurance Coverage: Early Release of Estimates From the National Health Interview Survey, January–March 2010 by Robin A. Cohen, Ph.D., and Michael E. Martinez, M.P.H., M.H.S.A., Division of Health Interview Statistics, National Center for Health Statistics. **IMPORTANT:** A footnote states, “A small number of persons were covered by both public and private plans and were included in both categories.” As a result, note that the percents in each column add up to greater than 100%.

Let S = persons in the U.S. under 65 years of age, and F = age under 18 years, and E = lacking health insurance coverage.

Are events E and F independent?

In Lecture 2.4 we were able to find values for $P(F)$ and $P(F^c)$, then put them into the tree diagram below. We used Bayes’ Theorem to determine $P(E)$.



From the table $P(E | F) =$

Conclusion:

Example H. The Shockingly Good Company has factories in Assateague, Betterton and Chestertown. The percentage of production and percentage of defective spark plugs made at each factory is given in the table below.

	Percent of Production	Probability of Defective
Assateague	45%	0.08%
Betterton	20%	0.4%
Chestertown	35%	0.1%

We'll put this information into a tree diagram.

What is the probability that a defective spark plug came from the Assateague factory? *answer: ≈ 0.2384*

What is the probability that a defective spark plug came from the Betterton factory? *answer: ≈ 0.5298*

What is the probability that a defective spark plug came from the Chestertown factory? *answer: ≈ 0.2318*

interpretation:

If the company is making decisions about where to focus efforts to improve quality, which factory should they choose? Why?