

## Stat 400, section 5.0 Chapter 5 preparation – Double Integrals

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We'll need double integrals for chapter 5. This section is a review (or possibly introduction for some) of how to evaluate them. There is also a supplement available from the Stat 400 page which has more practice for you.

Example A-1: Evaluate  $\int x^2 + 4xy + y^2 - 12y + 2 dx$ .

The principle here seems simple in concept, but is somehow difficult to implement for many people. The only “variable” is  $x$ , all other letters are treated like constants. *answer:*  $\frac{1}{3}x^3 + 2x^2y + xy^2 - 12xy + 2x + C$

Example A-2: Evaluate  $\int x^2 + 4xy + y^2 - 12y + 2 dy$  *answer:*  $x^2y + 2xy^2 + \frac{1}{3}y^3 - 6y^2 + 2y + C$

Example A-3: Evaluate  $\int_0^1 \int_1^2 x^2 + 4xy + y^2 - 12y + 2 dx dy$ .

The principle here is the same as for finding a mixed partial (second) derivative with respect to  $x$  then with respect to  $y$ . *answer:*  $\frac{5}{3}$

Example A-4: Evaluate  $\int_1^2 \int_0^1 x^2 + 4xy + y^2 - 12y + 2 dy dx$ . *answer:*  $\frac{5}{3}$

Example B: Find the double integral  $\iint_R x\sqrt{2x^2 + 3y} \, dy \, dx$  over the rectangular region  $0 \leq x \leq 1, 1 \leq y \leq 2$ .

This is really the same type of integral as in Examples A-3 and A-4. Be careful that you put the boundaries of integration into the correct integral. *answer:*  $\frac{1}{45} \left( 8^{5/2} - 5^{5/2} - 6^{5/2} + 3^{5/2} \right) \approx 1.1672$

Two notes on Example B:

1) We could have switched this around to do the more involved integration by substitution ( $dx$ ) first, as long as we kept the boundaries of integration straight.

$$\int_0^1 \int_1^2 x(2x^2 + 3y)^{1/2} \, dy \, dx = \int_1^2 \int_0^1 x(2x^2 + 3y)^{1/2} \, dx \, dy$$

However, it turns out to be a good bit harder if we do the switch. Rule of thumb: Do the easier integration first.

2) Integration by substitution and integration by parts won't show up much in this section. The probability density functions we'll encounter will be no more complicated than Examples A and B.

Example C: Find the volume under the surface  $z = \frac{1}{2xy}$  and above the rectangular region  $1 \leq x \leq e, 1 \leq y \leq e^2$ .

*answer:* 1

Example D: Find the volume under the surface  $z = e^{2x-3y}$  over the region  $R$ , bounded by  $0 \leq x \leq 1$ ,  $0 \leq y \leq 2$ .

answer:  $\frac{1}{6} (-e^{-4} + e^{-6} + e^2 - 1) \approx 1.0622$

Example E: Evaluate  $\int_0^1 \int_{-x}^{x^2} x^2 + 3xy + 2y \, dy \, dx$ . answer:  $\frac{23}{120}$

*We probably won't have time to get to this one in Lecture, and it's beyond the scope of the probability density functions we'll encounter in chapter 5. I include it for those of you who would like to play around with it for practice in evaluating double integrals.*