

Stat 401, section 9.2 Two Sample t Test and Confidence Interval

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In section 9.2, we're still comparing two groups, but the situation doesn't pass the "large-sample test", that is, either $m \leq 40$ or $n \leq 40$. So, instead of using Z as our test statistic we'll use T .

Basic assumptions:

Random variables $X_1, X_2, X_3, \dots, X_m$, are from a normal population distribution.

Random variables $Y_1, Y_2, Y_3, \dots, Y_n$, are from a normal population distribution.

The X and Y samples are independent of one another.

As with the large-sample case of section 9.1, we'll be considering the parameter $\mu_1 - \mu_2$. When we have two independent randomly-chosen large samples, we'll use the two sample means and two sample standard deviations (or variances) in our point estimates of expected value and standard error.

The formulas for confidence interval and test statistic for the small sample case will look just like those for the large-sample case. But, because we're relying on a t distribution, we need to specify degrees of freedom. This is where it starts to look complicated.

$$se_1 = \frac{s_1}{\sqrt{m}}, \quad se_2 = \frac{s_2}{\sqrt{n}}, \quad \nu = \text{largest integer less than } \frac{[(se_1)^2 + (se_2)^2]^2}{\frac{(se_1)^4}{m-1} + \frac{(se_2)^4}{n-1}}$$

9.1 Example A revisited – confidence interval: In doing a survey of textbook costs, Rodney found that at Whatsammatta U., in a sample of 19, the mean cost was \$63.42 with $s = \$8.13$. At Matriarch College (affectionately known as U. Mama) the mean textbook cost was \$67.19 in a sample of 23 with $s = \$7.29$. Construct a 95% confidence interval for the difference between the mean costs of textbooks at the two schools. (Previous studies support both populations being normally distributed.)

9.1 Example A revisited – hypothesis test: In doing a survey of textbook costs, Rodney found that at Whatsammatta U., in a sample of 19, the mean cost was \$63.42 with $s = \$8.13$. At Matriarch College (affectionately known as U. Mama) the mean textbook cost was \$67.19 in a sample of 23 with $s = \$7.29$. Test the hypothesis ($\alpha = 0.05$) that textbooks cost less at Whatsammatta U. than they do at U. Mama. (Previous studies support both populations being normally distributed.)

9.2 Example B: The Federal Trade Commission tests tires to rate them for wear. Data are amounts of tread remaining, measured in thousandths of an inch. At significance level $\alpha = 0.05$, do brand A tires perform differently than brand B? There is reason to believe that tire tread wear is normally distributed.

Brand						
A	125	64	98	38	90	106
B	133	65	103	37	102	115